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Seismic Stability of Elasto-Plastic Frames

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On the front-page: *Building in central Kobe with mid-height story collapse in the fault-normal direction (Kobe, Japan earthquake, January 17, 1995)*

*Photographer: unknown.*

Courtesy of Kobe Geotechnical Collection, Earthquake Engineering Research Center, University of California, Berkeley.
There is an extensive list of persons I am indebted with, since they supported, in one way or another, the development of this dissertation.

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To my family
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"Seismic stability of elasto-plastic frames" is an expression referring to the problem of collapse safety evaluation of ordinary structures subjected to strong earthquakes. It is a particular case of dynamic stability of structures. The latter expression covers a wide range of both theoretically and practically interesting problems of applied mechanics. Then, it is deemed useful to explicitly define the concept of seismic stability, which is characterised by its own peculiarities.

There have been in the past many discussions about the exact meaning of the words. What does exactly mean the expression 'dynamic stability'? As reported by Hoff (1965), Biezeno and Grammel explain that following Kirchhoff's definition, dynamics is the science of motion and forces, thus including statics, which is the study of equilibrium, and kinetics, which treats of the relationship between forces and motion. According to concepts taught at the author's University of Naples, these definitions should be slightly modified. In fact, the author was got used to think of kinetics as the science studying the laws of motion independently of the causes producing it. The relationship between forces and motion is instead the main concern of dynamics, which correctly includes statics as a particular case. On the other hand, as said by Hoff (1965), dynamics is generally accepted as the antonym of statics in everyday usage and this is the sense in which the word will be used in the current dissertation.

The most general and commonly accepted definition of stability is that given, at the author's knowledge, by Liapunov (1949). The rigorous mathematical formulation proposed by Liapunov will not be reported here, but the physical meaning of the Liapunov idea will be utilised. According to
Liapunov, a given equilibrium state (static or dynamic) is *stable* if and only if (by definition) it is possible to bound the response of the system within arbitrarily small amplitudes under arbitrarily chosen but appropriately small perturbations of the initial state. This concept is explained better with the help of Figure 1, referring to the case of an initial static equilibrium condition. After a small disturbance, oscillations originate but they are confined within a pre-selected, arbitrarily small, range. This indicates *stability* of that equilibrium condition. A particular case of stable response, not shown in the Figure, is that of *asymptotic stability*, where the system tends to return to its initial (undisturbed) condition after the action of the disturbance. On the contrary, the case of *instability* is shown in the form of 'divergence' in Figure 2 and of 'flutter' in Figure 3, adopting definitions established in the theory of aeroelasticity. In both the case of 'divergence' and 'flutter', the response of the system to a small perturbation is unbounded.

*Figure 1. Bounded response to a given disturbance.*

*Figure 2. Unbounded response to a given disturbance (divergence)*
Disturbance

Unbounded response (flutter)

Figure 3. Unbounded response to a given disturbance: flutter.

According to previous discussion, it is apparent that every stability problem is in principle a dynamic problem, since it involves the study of the dynamic response of the system to perturbations. In some particular cases, the dynamic aspect of the stability problem is only implicit and does not need to be faced. A very familiar case, for structural engineers, is that of the Euler buckling load of an axially compressed elastic column. In fact, the Euler approach consisted in finding, through the use of static equilibrium equations, another static equilibrium state in the immediate neighbourhood of the state whose stability is being investigated. The 'energy approach', descending more directly from the Liapunov definition of stability and being based on the Lagrange-Dirichlet theorem of stability, is fully equivalent to the Euler approach, in the sense that it does not require an explicit study of dynamics of the system being investigated. However, it is well known that several types of stability problems could not be discovered using a 'static approach'. Important examples are the case of 'parametric resonance', the case of 'impulsive loading', the case of 'follower forces' (or 'circulatory loads') and the case of aeroelastic problems. In all these cases, it is necessary to use the Newton's equations of motion to study the stability of a given (static or dynamic) equilibrium condition.

Therefore, the use of the expression 'dynamic stability', with reference to the seismic case, is strongly ambiguous, since it is not clear if the adjective 'dynamic' refers to the fact that external forces are dynamic ones or to the methodology used for investigating the problem (static or dynamic approach).
According to Hoff (1965), the definition of the stability problem given by Liapunov, basing on the idea of small disturbances, could be not always applicable in practical problems. An attractive example of such a case has been given by Hoff himself and is reported here for the reader. Consider a suitcase standing in an automobile. It is stable if the car goes on with a constant speed on a smooth road. But, whenever the driver brakes rapidly, the suitcase will fall flat on the floor. In this case, the small perturbation approach does not give an answer to the practical stability problem. Therefore, to cope with the exigencies of real life (using words of Hoff himself), a broad definition of stability has been proposed by the above Author and is reported here for the reader:

"A structure is in a stable state if admissible finite disturbances of its initial state of static or dynamic equilibrium are followed by displacements whose magnitude remains within allowable bounds during the required lifetime of the structure".

It should be recognised that the Liapunov concept of stability is addressed to solve a well-defined type of physical problems. This type is that one of studying the sensitivity of a given (static or dynamic) equilibrium solution to small variations in the theoretical initial conditions the solution has been mathematically derived from. A good example is the problem of studying the sensitivity of the trajectory of a missile to perturbations in the initial launching conditions. On the contrary the problem emphasised by Hoff is slightly different. To understand this difference, it is useful to think over the classical problem of the lateral stability of an axially compressed elastic column. The Euler buckling load individuates the maximum axial load under which the column is stable in the Liapunov sense. From a mathematical point of view, also an axial load equal to 90% of the buckling value is admissible, since the system is stable, in the Liapunov sense of small perturbations, under that load. However, it is apparent that a so small safety factor (1/0.9 = 1.1) could be practically not allowable, since there could exists, in the life-time of the structure, perturbations of the initial conditions (for example an increase of the axial load) that could produce the collapse of the structure. The same concept applies also to the example proposed by Hoff: the stability problem of the suitcase is actually a safety factor matter. Then, the stability problem emphasised by Hoff is actually of a type different from the original physical
concept intrinsic to the Liapunov definition. However, it is interesting to notice that the Hoff broad definition contains also the Liapunov concept of small perturbations.

The Hoff's definition of stability is very useful from the perspective of seismic engineering.

In fact, evaluating the 'Seismic Stability' of a given structure could be considered as the analysis of the ability of the structure to continue to serve, after the earthquake, the function it was designed for before the earthquake. Earthquakes produce damage to structures, thus reducing their ability to serve their functions. The degree of reduction of this ability is the inverse of the seismic stability. In other words, according to this point of view, the concept of seismic stability is not binary (structure stable or structure unstable), but rather there are different degree of stability. It is worth emphasising that, according to this definition, evaluating the seismic stability of a given building implies consideration of both structural and non-structural elements.

The measure of the degree of 'Seismic Stability' could be given by defining different performance levels (see Chapter 1 of this dissertation), which could be achieved during an earthquake. For example, if at the end of a given earthquake the degree of damage suffered by the building is so small that both structural and non-structural elements are generally in place, then the building is at its maximum degree of stability under that earthquake. On the contrary, when the earthquake is so strong to reduce to zero the structure ability to sustain gravitational loads, then the building is at its 'Seismic Stability Limit State', since no function can be served after the earthquake.

It is apparent that, in order to evaluate the safety of the structure against the Seismic Stability Limit State, the limit state itself must be previously identified, not only from a qualitative point of view, as previously made, but introducing engineering methods. This is the main objective of the current dissertation.

It is important to highlight that the seismic problem is of a double nature. In fact, in the classical problem of structural stability under static loads, the way of measuring the action intensity is well established: the entity of the applied forces is a well-known concept. On the contrary, in the seismic case, we have on one hand the need to identify the way of measuring the ground-motion shaking intensity, and on the other hand the problem of understanding
how to measure, quantitatively and objectively, the post-earthquake degree of stability of the structure. Paradoxically, the latter problem should be solved first. In fact, let us indicate by $S$ a generic ground motion parameter, which has been a-priori selected as a measure of the earthquake intensity. From the point of view of the evaluation of the safety against the Stability Limit State, the chosen parameter can be considered as the right one only if increasing it will reduce the degree of post-earthquake stability of the structure. Then, it is required to have previously defined how to measure the post-earthquake degree of stability of the structure. On the other hand, in order to measure the degree of stability (in the sense of the Hoff's problem of stability) requires knowing which is exactly the critical condition (in the sense of the Liapunov problem of stability). Then, identifying the critical post-earthquake structural conditions will allow us to achieve at the same time two objectives. First, the post-earthquake degree of stability of a damaged, but not collapsed, structure could be evaluated and, second, the earthquake shaking intensity measure and its critical value could be identified.

Finally, it is worth noting that, accordingly to previous discussion, the title of this dissertation should sound something like “Structural Stability of Elasto-Plastic Frames under Seismic Actions” or also “Seismic Stability Limit State of Elasto-Plastic Frames”. But, it has been decided to shorten the title using the definition ‘Seismic Stability’ in the strict sense of ‘Structural Stability under Seismic Actions’.
1.1 THE ‘PERFORMANCE-BASED’ APPROACH IN SEISMIC DESIGN

For many years structural engineers have got used to consider the design of an earthquake resistant structure as the design of a structure subjected to static lateral forces. Though practical and sometimes safe (if appropriate detailing rules are applied), this approach to the problem of seismic design inhibits, or at least makes it difficult, a clear understanding of the structure mechanical behaviour when subjected to earthquake actions (dynamic behaviour in the inelastic range).

Ensuring the stability of structures during strong ground motions in the context of current design practice is a rather complex issue. In fact, the strength and stability checks are carried out at fictitious force levels, under which the structure behaves elastically. Unfortunately, both at the global (structure) and at the local (element) level, stability is strongly affected by the inelastic deformation history developed during the earthquake. Perhaps, this is the reason why seismic codes do not directly deal with the stability problem.

Recently, several researchers have pointed out the need for a change in this perspective, emphasising the opportunity for developing new design and evaluation methodologies able to explicitly deal with the dynamic inelastic
response of the structure. The new method shall be based on a clear understanding of dynamics of structures and inelastic response of structural components. The main focus of many of these researchers has been the development of a general framework for research activities. The framework has been based on the philosophy that a modern earthquake resistant design should be able to graduate the seismic performance of the building in connection with the earthquake intensity, allowing the building owner to select 'performance objectives' to be pursued. A 'performance objective' is explicitly defined by ATC 40 (Applied Technology Council, 1996) as the 'desired building performance for a given level of earthquake ground motion'.

One main difference of the new 'performance based' earthquake engineering with respect to the traditional limit states philosophy is that in the latter methodology the building performance levels, the earthquake intensities and their combinations are pre-defined in a rigid format (the code format). On the contrary, the new approach provides a flexible format, which should allow the designer to meet the building owner choices, making the latter also aware of the real behaviour of the building to be expected in case of minor to more violent earthquakes. The new approach is based on general concepts that makes it applicable both in the design of new buildings and in the evaluation and retrofit of existing ones.

One main advantage of the new approach could be the explicit consideration of the true inelastic stability problem in the design/evaluation procedure. The global structural stability problem, which is the main concern of this dissertation, appears to be as just one of the aspects to be considered within the performance-based approach. It is a particular structural performance level to be checked under appropriately chosen earthquake intensities. This is discussed in the following paragraphs, where the 'Structural Stability' performance level, as conceived by modern earthquake engineering, is introduced.

A building performance level describes a building damage condition. The latter is the combination of a 'non-structural' damage, which is damage to non-structural systems, and a 'structural' damage, which is damage to the main load carrying systems. Among the proposed descriptions of structural and non-structural limiting damage conditions, the ones proposed by ATC 40 are considered here for discussion. Six structural and five non-structural
performance levels are introduced in the cited document, and are briefly recalled here with the relevant qualitative definitions:

- Structural Performance Levels:

  Immediate Occupancy: The post-earthquake damage state in which only very limited structural damage has occurred; the basic vertical and lateral force resisting systems of the building retain nearly all of their pre-earthquake characteristics and capacities.

  Damage Control (range): This term is actually not a specific level but a range of post-earthquake damage states ... from Immediate Occupancy to Life Safety.

  Life Safety: The post-earthquake damage state in which significant damage to the structure may have occurred but in which some margin against either total or partial structural collapse remains. ... It should be expected that extensive structural repairs will likely be necessary prior to reoccupation of the building, although the damage may not always be economically repairable.

  Limited Safety (range): This term is not a specific level but a range of post-earthquake damage states that are less than Life Safety and better than Structural Stability.

  Structural Stability: This level is the limiting post-earthquake structural damage state in which the building's structural system is on the verge of experiencing partial or total collapse. ... However, all significant components of the gravity load resisting system continue to carry their gravity demands. ... It should be expected that significant major structural repair will be necessary prior to reoccupancy.

  Not Considered: This is not a performance level, but provides a placeholder for situations where only nonstructural seismic evaluation or retrofit is performed.
- Nonstructural Performance Levels:

Operational: The post-earthquake damage state in which nonstructural elements and systems are generally in place and functional.

Immediate Occupancy: The post-earthquake damage state in which nonstructural elements and systems are generally in place.

Life Safety: This post-earthquake damage state could include considerable damage to nonstructural components and systems but should not include collapse or falling of items heavy enough to cause severe injuries either within or outside the building. Secondary hazards from breaks in high-pressure, toxic, or fire suppression piping should not be present.

Hazards Reduced: This post-earthquake damage state could include extensive damage to nonstructural components and systems but should not include collapse or falling of large and heavy items that could cause significant injury to groups of people, such as parapets, masonry exterior walls, or large, heavy ceilings.

Not Considered: Nonstructural elements, other than those that have an effect on structural response, are not evaluated.

As already said, the combination of structural and non-structural performance levels gives rise to building performance levels. Table 1.1.1.1 (ATC 40) summarises some possible building performance levels in seismic areas.

To form a performance objective a given performance level must be combined with a given earthquake ground motion level. Then, it is necessary to define also different levels of earthquake ground shaking intensities. There are different proposals about these levels, too. However, three levels are commonly considered, as summarised here (definitions are taken again from ATC 40):

- The Serviceability Earthquake: Ground motion with a 50 percent chance of being exceeded in a 50-year period.
- The Design Earthquake: *Ground motion with a 10 percent chance of being exceeded in a 50-year period.*

- The Maximum Earthquake: *The maximum level of ground motion expected within the known geologic framework due to a specified single event (median attenuation) (deterministic approach), or the ground motion with a 5 percent chance of being exceeded in a 50-year period.*

Sometimes, the three earthquake ground motion levels are called 'frequent', 'rare' and 'very rare', respectively. Besides, the 'very rare' ground motion shaking is often considered as one having a 2 percent (instead of 5) probability of being exceeded in 50 years (Somerville et al., 1997).

Usually, the Design Earthquake is combined with the building performance level called Life Safety and the Serviceability Earthquake is combined with one of the two building performance levels called Operational and Immediate Occupancy. Moreover, the Maximum Earthquake is often combined with the Structural Stability performance level. This is shown in Table 1.1.1.2.

<table>
<thead>
<tr>
<th>Table 1.1.1.1. Building performance levels</th>
</tr>
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<tbody>
<tr>
<td>Building performance levels</td>
</tr>
<tr>
<td>Nonstructural Performance Levels</td>
</tr>
<tr>
<td>Structural Performance Levels</td>
</tr>
<tr>
<td>SP-1 Immediate Occupancy</td>
</tr>
<tr>
<td>SP-2 Damage Control (range)</td>
</tr>
<tr>
<td>SP-3 Life Safety</td>
</tr>
<tr>
<td>SP-4 Limited Safety (range)</td>
</tr>
<tr>
<td>SP-5 Structural Stability</td>
</tr>
<tr>
<td>SP-6 Not Considered</td>
</tr>
</tbody>
</table>

| NP-A Operational  | 1-A Operational | 2-A | NR | NR | NR | NR |
| NP-B Immediate Occupancy | 1-B Immediate Occupancy | 2-B | 3-B | NR | NR | NR |
| NP-C Life Safety | 1-C | 2-C | 3-C Life Safety | 4-C | 5-C | 6-C |
| NP-D Hazards Reduced | NR | 2-D | 3-D | 4-D | 5-D | 6-D |
| NP-E Not Considered | NR | NR | 3-E | 4-E | 5-E Structural Stability | Not Applicable |

*Legend: NR = Not recommended*
Until now the description of building performance levels has been mainly made in a qualitative sense. This is useful for the owner, which is not a technician, but it makes difficult for the engineer to translate the owner desire into an operating technical solution. Therefore, building performance levels must be related to building limit states, which can be quantified through engineering methodologies. Analogously, the term 'earthquake ground motion level' is quite intuitive from the qualitative point of view, but it is actually difficult to define the level of ground shaking intensity from a quantitative point of view. The general framework of performance based earthquake engineering, as briefly presented here, only gives a general idea of the requirements to be satisfied in a modern design or evaluation procedure. Also, some criticism could be raised to the definition of each structural and non structural performance level, to their combination presented in Table 1.1.1.1 for defining building performance levels, to the ground motion levels introduced, to the 'usual' combination of performance and ground motion levels for defining the performance objectives. However, this is a philosophical matter rather than an engineering one. But it is essential, for making the 'performance based' methodology actually applicable in the daily practice, to clearly define the mechanical parameters that should be checked and their limit values corresponding to the achievement of a given limiting damage condition. In other words, the structure limit states associated with each performance level are to be defined on one hand and the ground motion parameters controlling the achievement of the limit states are to be selected on the other hand.
1.2 MOTIVATION AND SCOPE OF THE STUDY

Current codified design philosophy for structures in seismic zones is long far from the noble concept of 'performance based' earthquake engineering. Many questions could be raised at the light of the discussion in the previous section. For example:

1. The serviceability and ultimate limit states, which are defined in modern seismic codes (such as the European EC8 (CEN, 1994)), to which performance levels do correspond?
2. Which is the earthquake ground motion level assumed by seismic codes, such as Eurocode 8 (CEN, 1994), in the serviceability limit state checking?
3. Which is the safety at collapse (meaning dynamic instability) of a code-designed building structure?

The answer to the first question cannot be complete and exhaustive. First of all, in the performance based methodology multiple building performance levels are defined under the serviceability ground motion level. The serviceability limit state could therefore be potentially corresponding to both Operational and Immediate Occupancy damage limiting conditions. Besides, the ultimate limit state should correspond to some performance in the Damage Control range. Theoretically, it should be correspondent to the Life Safety performance level.

The answer to the second question is more troublesome. In fact, in the European seismic code EC8 (CEN, 1994) it is clearly specified that the ultimate limit state requirement should be satisfied under an earthquake ground motion level having a 475 years return period. Assuming for simplicity a uniform distribution of probability over time, the probability of exceedance of the relevant earthquake in 50 years is given by $\frac{50}{475} \approx 0.10$ (10%), which correspond to the Design Earthquake ground motion level. On the contrary, for checking the serviceability limit state, EC8 does not explicitly indicate the relevant earthquake intensity, but uses a quite involved and indirect procedure. This procedure is not explicitly discussed here, but will be deepened in Chapter 5 when discussing the results of some numerical analyses carried out for this dissertation. It is sufficient here to notice that the earthquake intensity considered by EC8 for the serviceability limit state checking could be presumed to be around $\frac{1}{3}$ to $\frac{1}{2}$ of the earthquake
intensity used for the ultimate limit state checking. It is impossible, from this information, to know the probability of exceedance used by EC8 for defining the serviceability ground motion level.

The third question is strictly related to the contents of the current dissertation. The attempt to answer that question can be considered as the main motivation of the study itself. As previously noted, current codified design rules do not explicitly deal with collapse of structures, since safety against dynamic instability is not explicitly checked. In fact, modern codes try to guarantee an adequate safety against collapse of structures subjected to strong earthquakes in an indirect manner, i.e. by prescribing minimum levels of structural strength compatible with a target level of ductility demand. This target has mainly the meaning of an upper bound over which strength degradation is likely to take place. It is contended that this methodology is strictly related to the type of hysteresis model that can be today adopted with sufficient reliability, i.e. the elastic – perfectly plastic type of model. In fact, such a model is unable to represent degradation phenomena related to cyclic inelastic actions. Consequently, the ultimate limit state is practically identified with the achievement of a limiting damage condition over which the validity of the mathematical model adopted in the numerical analyses is lost (i.e. significant degradation takes place). Such an approach is, in principle, adequate to guarantee that the structure collapse will occur for seismic intensities greater than those stipulated by the code with reference to the conventionally defined ultimate limit state and for the given seismic zone. However, the ‘real’ safety at collapse of the structure remains unknown. It is intuitive that the only way to obtain a reliable prediction of safety at collapse of the whole structure is by developing more complete hysteresis models, able to take account of degradation phenomena. Despite it is apparent, from a physical point of view, that collapse under earthquake actions means dynamic instability, reliable methods for predicting the occurrence of this phenomenon do not still exist. Many papers in the technical literature, dealing with the problem of global damage evaluation, testify the need of a clear methodology for evaluating the dynamic stability limit condition. In fact, global structural damage is frequently measured by means of ‘global damage indices’ lacking of physical meaning and also misleading about the actual damage in some cases (Ghobarah et al., 1999). Even when the attention is shifted towards a
more physically meaningful dynamic stability limit condition (Bernal, 1992),
the limit state is not yet well identified, being described in a generic manner as
an asymptotic situation achieved when increasing the peak ground
acceleration.

Therefore, despite the 'Structural Stability' performance level is just one of
the structural damage condition to be checked, according to a modern
earthquake-resistant design practice, it is contended that collapse safety
evaluation is still the more demanding challenge of modern earthquake
ingineering. In addition, the definition of the Stability Limit State for a
building subjected to seismic actions has not yet been clearly set up.

Then, the main scope of the current dissertation is to deal with 'dynamic
stability' of structures subjected to strong earthquakes, aiming at setting up a
clear methodology for identifying the limit state.
Chapter 2
Seismic Stability of SDoF Systems

2.1 INTRODUCTION

Single-degree of freedom (SDoF) systems are, perhaps, the most popular and extensively studied structural types in earthquake engineering and structural dynamics context. Their mechanical behaviour has been deeply investigated over the years. SDoF systems are the basis for understanding the elastic dynamics of multi-degree of freedom (MDoF) structures, based on the concept of response spectrum, first introduced by Benioff (1934) and Biot (1941). Many codified design rules of real structures are based on results concerning SDoF systems. It is deemed that this popularity is related to the simplicity of the model, which allows a clear understanding of its mechanical behaviour from the physical point of view.

With reference to the problem of seismic stability of SDoF systems, the paper by Jennings and Husid (1968) is pioneering and poses the basis for a clear understanding of the conditions that lead to dynamic instability under strong earthquakes. The above Authors studied the influence of gravity loads on the seismic response of the SDoF system depicted in Figure 2.1.1. According to Jennings and Husid, the transition from vibration to the exponential growth of the lateral drift occurs when a critical value of the maximum rotation is achieved. The critical rotation is identified as that one for which the destabilising 'second-order' moment produced by gravity loads


$(W\phi cL)$ is balanced by the ‘first-order’ plastic strength $(M_y)$ (see Figure 2.1.1 for the meaning of symbols).

![Diagram of an elementary model for studying the seismic stability of elasto-plastic systems](image)

**Figure 2.1.1. An elementary model for studying the seismic stability of elasto-plastic systems (adapted from Jennings & Husid 1968).**

A very interesting paper, not directly concerned with the dynamic stability problem but having important consequences in understanding the conditions leading a structure to collapse under an earthquake, is that of McRae e Kawashima (1997). In the paper, Authors clarify the main role of a negative stiffness in increasing the residual deformation of the structure, i.e. the permanent deformation produced by the earthquake. In the following, the main role of residual displacements for the identification of the dynamic stability limit state will become clear.

The brilliant intuition of Jennings and Husid will be deeply utilised within this dissertation, particularly in the current Chapter dealing with the seismic stability of SDoF systems. However, the Jennings and Husid criterion will be deeply investigated and reviewed, concluding that the stability limit state is not exactly identified by the condition $\Delta_{\text{max}} = \Delta_c$, but rather by the condition $\Delta_r = \Delta_c$, $\Delta_c$ being the residual lateral displacement of the structure after the earthquake. The main conclusions coming out from the study of SDoF systems will be then generalised in Chapter 3, applying them to the study of the seismic stability of MDoF structures.

To start the study, a prototype SDoF system is to be chosen. The choice has been made taking in mind that some peculiar features of the seismic behaviour of buildings, such as the so-called P-Δ effects, must be highlighted by the
model. Moreover, a simplified system, with respect to the one chosen by Jennings and Husid, has been adopted, with the aim of obtaining simple closed-form analytical relationships. Thus, the SDoF system depicted in Figure 2.1.2 has been selected. In Figure 2.1.2, the internal resisting base moment variation \( M \) has been indicated as a function of \( \phi \), which is the base spring rotation, of \( \phi_\text{inc} \), which is the base spring incremental rotation, and also of a number of generic state variables, \( \alpha_1, \alpha_2, \ldots \), which are parameters allowing for consideration of damage phenomena related to plastic deformations. However, for the sake of clarity, discussion will start assuming that the base spring is characterised by ideal hysteresis behaviour, such as that based on the elastic – perfectly plastic moment–rotation relationship (Section 2.2). Thus, local damage phenomena will be firstly neglected. A generalisation of concepts introduced in Section 2.2 will be presented in Sections 2.3 and 2.4, with reference to strength-degrading hysteresis models. In particular, the effect of degradation of mechanical properties of the base spring owing to excessive plastic deformation will be deeply studied in Section 2.3, showing the formidable importance of strength degradation in determining the lateral stability limit state of the structure.

\[ M = M(\phi, \phi_\text{inc}, \alpha_1, \alpha_2, \alpha_3, \ldots) \]

*Figure 2.1.2. The examined SDoF prototype.*
2.2 SEISMIC STABILITY BASED ON THE ELASTIC – PERFECTLY PLASTIC HYSTERESIS MODEL

In this Section, it will be shown that seismic stability (dynamic elasto-plastic stability) can be studied through a 'static' approach, very similarly to what is usually done for checking the Euler buckling stability. Thus, the dynamic stability of static elasto-plastic equilibrium configurations of the SDoF system shown in Figure 2.1.2 will be firstly investigated, looking for the existence of critical values of the lateral displacement of the structure. Then it will be shown that dynamic instability during earthquakes occurs when the static critical value of the displacement is achieved.

Static equilibrium, in the laterally displaced configuration of the SDoF system shown in Figure 2.1.2, writes:

\[
\Delta - \Delta_{LM} = \frac{P \Delta}{L}
\]  

(2.2.1)

where the small displacement approximation \( \varphi = \frac{\Delta}{L} \) has been made. The meaning of the terms in equation (2.2.1) is clearly shown in Figure 2.1.2.

The second term on the right hand side of equation (2.2.1) is what is usually called the 'P-\(\Delta\) effect', which has been made linear through the small displacements approximation. \( H \) indicates a generic external force acting at the top of the cantilever. Equation (2.2.1) allows us to trace the equilibrium path of the cantilever when its lateral displacement is monotonically increased. The external lateral force that guarantees equilibrium at any stage of lateral displacement will be called the 'external restoring (lateral) force' or, simply, the 'restoring (lateral) force'. It is then considered already affected by the P-\(\Delta\) effect. On the contrary, the force given by \( \frac{M(\varphi)}{L} = \frac{M(\Delta/L)}{L} \) is here called the 'internal restoring (lateral) force' or, also, the 'first order restoring (lateral) force'. It is unaffected by the P-\(\Delta\) effect.

The system mechanical behaviour depends upon the \( M(\varphi) \) relationship of the base spring, which is assumed here to be the elastic – perfectly plastic moment – rotation relationship (Figure 2.2.1). Correspondingly, the relevant
$H-\Delta$ relationship, derived on the basis of the equilibrium condition (2.2.1), is as shown in Figure 2.2.2. The case under examination is quite popular and well known (see, for example, Mazzolani and Piluso 1996). Therefore, a complete derivation of the $H-\Delta$ relationship and the relevant mathematical details are here omitted.

**Figure 2.2.1. The monotonic moment – rotation relationship considered.**

**Figure 2.2.2. The monotonic lateral force vs. lateral displacement relationship in case of elastic - perfectly plastic moment vs. rotation response of the base spring.**
According to the Jennings and Husid criterion, dynamic instability during earthquakes occurs when the lateral displacement demand is equal to $\Delta_c$, which is the zero-strength lateral displacement, as shown in Figure 2.2.2. In fact, when $H$ is set equal to zero, the equilibrium condition 2.2.1 gives us the following equation determining the critical value of the lateral displacement:

$$\frac{P}{L} \Delta_c = \frac{M_y}{L}$$

(2.2.2)

which has the same meaning of the criterion suggested by Jennings and Husid (‘second-order’ destabilising moment equal to the ‘first-order’ stabilising one). $\Delta_c$ will be called the 'collapse displacement' or, also, the 'critical displacement'. For $\Delta > \Delta_c$ the destabilising external moment related to gravity loads is greater than the stabilising internal one. Thus, when the structure is displaced laterally of $\Delta_c$ an indefinitely small increment of the lateral displacement itself cannot be balanced just by the internal strength but requires a stabilising negative (with a sign opposite to that of displacement) force. Though it is quite intuitive that the structure displaced laterally of $\Delta_c$ is dynamically unstable, it will be shown here that this instability could be rationally demonstrated through a simple reasoning. To this end, the free oscillations of the examined single-degree of freedom system with geometrical and mechanical non-linearity will be briefly discussed assuming that all the mass is lumped at the top of the rigid bar, so that to study an 'inverted pendulum' system as shown in Figure 2.2.3.

The dynamic equilibrium equation of the SDoF system, accounting for second order geometrical effects, mechanical non-linearity and linear viscous damping, writes as follows:

$$(-m\ddot{x}_g - m\dddot{x} - b\dot{x}) L - M + P\Delta = 0$$

(2.2.3)

where $\ddot{x}_g$ is the ground acceleration.

Neglecting the viscous damping force, i.e. assuming:

$$H = -m(\ddot{x}_g + \dddot{x}) b\dot{x} \approx -m(\ddot{x}_g + \dddot{x})$$

(2.2.4)
and dividing equation (2.2.3) by $m$ and $L$, we can write:

\[
(-\ddot{\Delta} - \ddot{\Delta}) \cdot \frac{M}{mL} + \frac{P}{mL} \Delta = 0
\]

(2.2.5)

Isolating the inertia force related to the ground acceleration, one gets:

\[
-\ddot{\Delta}_g = \frac{M}{mL} + \ddot{\Delta} - \frac{P}{mL} \Delta
\]

(2.2.6)

\[\text{Figure 2.2.3. The examined single-mass single-degree of freedom system.}\]

With an elastic – perfectly plastic type of moment – rotation relationship of the base spring, we can distinguish two different ranges of behaviour: the elastic range and the perfectly plastic range of oscillations. Mathematically, equilibrium in the two different ranges is represented by the two following differential equations:

\[
M = k \frac{\Delta}{L} \text{ (elastic behaviour)} + \ddot{\Delta}_g = 0 \text{ (free oscillations)} \Rightarrow \\
\Rightarrow \ddot{\Delta} = \left( k \frac{1}{mL^2} - \frac{P}{mL} \right) \Delta
\]

(2.2.7)
According to equation (2.2.7), during free oscillations of the mass with an elastic behaviour of the base spring, a straight line (indicated as line (1) in Figure 2.2.4) represents the $\Delta - \ddot{\Delta}$ equilibrium path. Analogously, equation (2.2.8) indicates that two parallel straight lines represent the free oscillations of the mass $m$ when the base spring is fully yielded under negative or positive bending moments (see lines (2) and (2') in Figure 2.2.4). It is useful to notice that the intersection of the straight line (2) with the horizontal axis ($\dot{\Delta} = 0$ in equation (2.2.8)) is achieved with $\Delta = \Delta_c = M_y/P$, which is the critical displacement computed according to equation (2.2.2).

Figure 2.2.4. Graphical representation of the equilibrium paths during free oscillations of the SDoF system represented in figure 2.2.3 and equipped with the elastic – perfectly plastic moment – rotation relationship of the base spring.
Now let us consider a system that starts to move in one direction from its initial configuration \((\Delta = 0, \dot{\Delta} = 0)\), due to an initial imposed velocity \(\dot{\Delta} > 0\). For \(\Delta > 0\) and \(\Delta < \Delta_y\), with \(\Delta_y\) indicated in Figure 2.2.4, the mass is decelerating \((\ddot{\Delta} < 0)\) thanks to the increasing elastic reaction of the base spring (see Figure 2.2.5, branch (1)). For \(\Delta > \Delta_y\) this deceleration is decreasing and approaches zero when \(\Delta = \Delta_c\). At this point equilibrium is unstable. In fact, for \(\Delta > \Delta_c\) equilibrium is possible only with \(\dot{\Delta} > 0\), that means the system is accelerating in the same direction of the lateral displacement. So the system tends to move definitely away from its initial configuration. If the difference between the initial kinetic energy and the maximum elastic strain energy (i.e. strain energy associated to the yield point) can be dissipated by the plastic part of the lateral deformation, then the system will stop before reaching the unstable state. On the contrary the system will reach \(\Delta_c\) and, therefore, it will collapse (see Figure 2.2.5).

![Figure 2.2.5. Unbounded response of the SDoF system when it reaches the lateral displacement \(\Delta_c\).](image-url)
Actually, viscous damping is not zero, as previously assumed. Since damping forces are stabilising ones, it could arise the doubt that $\Delta_c$ is not rigorously a stability limit point. To remove immediately this doubt, the following fundamental observation could be made. Let us write again the dynamic equilibrium equation, introducing also viscous damping forces, as shown hereafter:

\[
\ddot{\Delta} = -\frac{b}{m} \dot{\Delta} - \frac{M_y}{mL} + \frac{P\Delta}{L}
\]  
(2.2.9)

The algebraic sum of the second and third terms on the right hand side of equation (2.2.9) is zero for $\Delta = \Delta_c$ and positive for $\Delta > \Delta_c$, as previously shown. Let us indicate this algebraic sum as $H_s$, thus re-writing equation (2.2.9) as follows:

\[
\ddot{\Delta} = -\frac{b}{m} \dot{\Delta} + H_s
\]  
(2.2.10)

The first term on the right hand side of equation (2.2.10) is negative and increasing in absolute value when increasing the velocity of the mass, with a positive sign. Considering that the term $H_s$ has a very simple linear variation with $\Delta$, the differential equation (2.2.10) could be integrated over time, thus obtaining the time-history of the mass displacements and verifying if they are divergent. However, the dynamic instability of the static equilibrium condition $\Delta = \Delta_c$ can be argued in a simpler manner. In fact, let us consider the mass starting from $\Delta = \Delta_c$, with a small impressed positive velocity $\dot{\Delta} > 0$. Besides, let us assume, ab absurdo, that the mass will stop ($\dot{\Delta} = 0$) at a given time, owing to the stabilising effect of the damping force. At that time, the dynamic equilibrium condition (2.2.10) says that $\dot{\Delta} > 0$, since $H_s > 0$. Then, the mass will be accelerating in the same direction of the positive displacement and will move definitively away from the initial static equilibrium configuration (divergence).

Applying the concept expressed in Chapter 1 (Section 1.3), the system seismic stability should be judged based on its residual lateral displacement after the earthquake. In fact, if $\Delta_e < \Delta_c$, then the structure is left by the
earthquake in a static equilibrium condition that can be considered stable form a dynamic point of view, as shown in the previous discussion. On the contrary residual lateral displacements $\Delta_r > \Delta_c$ are impossible, since, as previously demonstrated, the system will tend to move definitively away from its static equilibrium state, also when a very small lateral velocity is applied to it (dynamically unstable in the Liapunov sense).

Now, let us indicate by $S$ a generic parameter measuring the ground motion shaking intensity. Let be $S_c$ the minimum value of $S$ inducing a residual lateral displacement greater than or equal to $\Delta_c$. $S_c$ could be defined as the critical earthquake intensity. In fact, under the earthquake having intensity equal to $S_c$ the system residual equilibrium configuration is a stability limit state. If (and only if) a monotonic relationship could be established between $S$ and $\Delta_r$, then it could be stated that earthquake intensities greater than $S_c$ are impossible. In this case the concept of critical earthquake intensity is analogous to the concept of critical load for static actions. However, this is a different problem with respect to the one faced until now. In fact, definition of $S$ involves, first of all, the understanding of the relationship between the ground motion parameters and deformation demand. The objective of the current dissertation, which has been partially fulfilled until now, is the characterisation of the dynamic stability limit state, independently of the external action. The ‘critical’ lateral displacement has been defined from the viewpoint of structural behaviour, without any concern of the earthquake action. Any earthquake inducing a residual permanent post-earthquake deformation equal to $\Delta_c$ is a critical earthquake. Definition of the limit state is the first step towards the understanding of the conditions leading to the achievement of that limit state. The problem of measuring the earthquake intensity will be empirically faced in Chapter 5, where several numerical analyses will be performed using a well-known ground motion scaling technique, based on the assumption of the peak-ground acceleration as main damage potential measure. As it will be shown, this assumption could not always yields a monotonic relationship between the earthquake scaling factor and the residual deformation of the structure. In any case, the minimum value of the selected ground motion intensity parameter inducing a residual deformation equal to $\Delta_c$ could be taken as a limiting critical value of the earthquake intensity.
Until now, the dynamic instability of the static equilibrium configuration $\Delta = \Delta_c$ has been shown. During an earthquake the system configuration characterised by a lateral displacement equal or greater than $\Delta_c$ is a dynamic equilibrium state. The relevant equilibrium equation must be written considering the ground acceleration (see equation 2.2.3). Based on this equilibrium equation, it will be hereafter shown that it is possible for the maximum lateral displacement of the mass to exceed $\Delta_c$ and then come back to a residual displacement less than $\Delta_c$, depending on the ground acceleration time-history. In other words, the following statement will be demonstrated:

$$\exists \ddot{\Delta}_g : \Delta_{\text{max}} > \Delta_c ; \Delta_r < \Delta_c$$

(2.2.11)

which means the possibility to have ground acceleration time-histories implying a maximum displacement demand greater than $\Delta_c$ but a residual displacement less than $\Delta_c$, thus leaving the structure in a stable post-earthquake state.

In order to demonstrate the statement (2.2.11), it is useful to reconsider the lateral force vs. lateral displacement relationship, taking into account now also the range $\Delta > \Delta_c$. Equilibrium is possible in this range if a negative lateral force is applied to the system, as previously said (see Figure 2.2.6, point of co-ordinates $(\Delta_1, H_1)$). Viscous damping and inertia forces could give this negative force in the dynamic range of behaviour. In fact, dynamic equilibrium at $\Delta_1$ can be written as follows:

$$-m\ddot{\Delta}_g - m\ddot{\Delta}_1 - b\dot{\Delta}_1 = \frac{M_1}{L} - \frac{P}{L} \Delta_1$$

(2.2.12)

If at the time $t_1$, with a displacement $\Delta_1$, the mass will invert its movement then $\dot{\Delta}_1 = 0$, $\ddot{\Delta}_1 < 0$. From equation (2.2.12), the mass acceleration at the same time $t_1$ is given by:

$$\ddot{\Delta}_1 = -\ddot{\Delta}_g - \frac{M_1}{mL} + \frac{P}{mL} \Delta_1$$

(2.2.13)
It is always possible to have a ground acceleration time-history ($\ddot{A}_g$) such that the algebraic sum on the right hand-side of equation (2.2.13) is negative, thus effectively implying an inversion of the movement of the mass ($\ddot{A}_1 < 0$). Then, it cannot be excluded that the system will survive that special earthquake by following the lateral force vs. lateral displacement pattern illustrated in Figure 2.2.6.

Based on the previous simple reasoning, it must be concluded that the condition $\Delta_{\text{max}} > \Delta_c$ is not a sufficient condition for dynamic instability of elasto-plastic systems during earthquakes.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.2.6.png}
\caption{Possible survival of the system when it is displaced laterally of $\Delta > \Delta_c$ by a ground acceleration time-history.}
\end{figure}

However, it is to be noted that a system displaced laterally of $\Delta_{\text{max}} = \Delta_c$ during an earthquake is very likely to collapse (to result in an unbounded response) during the earthquake. The reason is discussed in the next paragraphs.

Let us re-consider the monotonic loading conditions firstly examined to derive the $H-\Delta$ equilibrium path. If the structure is displaced laterally of $\Delta_{\text{max}}$ and then unloaded ($H = 0$), a residual lateral displacement $\Delta_r$ will remain, owing to the plastic part of the lateral displacement. When the maximum displacement approaches $\Delta_c$, then the residual displacement also approaches...
$\Delta_c$ and vice versa (see Figure 2.2.7). Therefore, the following equivalence holds true in case of a monotonically increasing lateral displacement:

$$\Delta_{\text{max}} \rightarrow \Delta_c \iff \Delta_r \rightarrow \Delta_c$$

(2.2.14)

\[ \begin{array}{c}
\Delta_{\text{max}} \\
\Delta_r \\
\Delta_c \\
\Delta_{\text{cr}} \\
\Delta_{\text{max}} \\
\end{array} \]

Critical displacement: $\Delta_{\text{max}} = \Delta_r = \Delta_c$

Figure 2.2.7. At the critical static condition the maximum and the residual lateral displacement are coincident.

In other words, the critical condition, in case of static and monotonic increase of the lateral displacement, could be written as follows:

$$\Delta_{\text{max}} = \Delta_r = \Delta_c$$

(2.2.15)

In the paper by McRae and Kawashima (1995), which has been already cited in the introduction, the Authors explain that residual displacements are increasing with an increasing earthquake intensity. More precisely, they argument that an increase of the earthquake intensity, producing an increase of the maximum lateral displacement exceeding the point of maximum static lateral strength, will produce also an increase of the residual displacement. The reason is the global negative stiffness of the structure, which produces an asymmetric yielding strength, with a lower bound in the direction where the negative stiffness has been activated. Thus, plastic deformations tend to accumulate in the same direction where the negative stiffness has been activated and, therefore, the residual displacement tends to increase. It must be recognised that it is not clear what should be taken as earthquake intensity and
what it exactly means the expression 'tends to increase'. The validity of
condition (2.2.14) and, equivalently, of condition (2.2.15), as an empirical
stability criterion in case of earthquake ground motions, will be experimented,
through numerical analyses, in Chapter 5. As it will be shown, in case of bi-
linear hysteresis behaviour of plastic hinges, the difference between \( \Delta_{\text{max}} \) and
\( \Delta_r \) tends to decrease and becomes very small at collapse, which is identified by
the condition \( \Delta_r = \Delta_c \). Then, from a practical point of view, the condition \( \Delta_{\text{max}} = \Delta_c \) is equivalent, in this case to the condition \( \Delta_r = \Delta_c \).

2.3 SEISMIC STABILITY BASED ON DEGRADING HYSTERESIS MODELS

2.3.1 The effect of monotonic strength degradation

In order to show the effect of monotonic strength degradation on the seismic
stability limit state, let us assume firstly a tri-linear moment – rotation relationship (Figure 2.3.1.1), which can be considered a generalisation of the elastic – perfectly plastic model discussed in the previous section.

![Figure 2.3.1.1 The monotonic moment – rotation relationship considered.](image)

The mathematical formulation of the tri-linear \( M(\phi) \) curve may be given by:

\[
M(\phi) = (k_0 - k_h)\phi_{e1} + (k_h - k_s)\phi_{e2} + k_s\phi
\]  

(2.3.1.1)
where, obviously, $\Delta_{e1} = \phi_{e1}L$ and $\Delta_{e2} = \phi_{e2}L$. Relationship (2.3.1.4) has the advantage to be valid for every value of $\Delta$, if appropriate values of $\Delta_{e1}$ and $\Delta_{e2}$ are selected. In particular, we can distinguish three ranges of behaviour, corresponding to the elastic ($k_0$), plastic-hardening ($k_h$) and plastic-softening ($k_s$) behavioural phases of the base spring. Mathematically, the relevant $H(\Delta)$ relationship is as follows:

1. If $\Delta \leq \Delta_y$, then $\Delta_{e1} = \Delta_{e2} = \Delta \Rightarrow$

   $$H = \left(\frac{k_0 - k_h}{L^2}\right)\Delta_{e1} + \left(\frac{k_h - k_s}{L^2}\right)\Delta_{e2} + \left(\frac{k_s}{L^2} - \frac{P}{L}\right)\Delta$$

   (2.3.1.5)

2. If $\Delta_y \leq \Delta \leq \Delta_s$, then $\Delta_{e1} = \Delta_y$, $\Delta_{e2} = \Delta \Rightarrow$

   $$H = \left(\frac{k_0 - k_h}{L^2}\right)\Delta_y + \left(\frac{k_h}{L^2} - \frac{PL}{k_h}\right)\Delta$$

   (2.3.1.6)
\[ \text{if } \Delta \geq \Delta_s \quad \text{then} \quad \Delta_{e1} = \Delta_y \quad ; \quad \Delta_{e2} = \Delta_s \quad \Rightarrow \quad H = \left( \frac{k_0 - k_h}{L^2} \right) \Delta_y + \left( \frac{k_h - k_s}{L^2} \right) \Delta_s + \frac{k_s}{L^2} \left( 1 - \frac{PL}{k_s} \right) \Delta \]

(2.3.1.7)

where \( \Delta_y = \varphi_y L \) and \( \Delta_s = \varphi_s L \). The tri-linear \( M(\varphi) \) relationship is then transferred to the \( H(\Delta) \) relationship, but the slope of the three branches have to be modified due to the \( P-\Delta \) effect. In particular, the first order elastic stiffness is reduced by means of the non-dimensional factor \( (1 - p) = (1 - PL/k_0) \). In the same way, stiffness during the plastic-hardening and plastic-softening ranges is reduced by the analogous coefficients \( (1 - PL/k_h) \) and \( (1 - PL/k_s) \), respectively. It is interesting to notice that \( p = 1 \) corresponds to a zero initial lateral stiffness, i.e. to a vertical loading level equal to the elastic critical Euler load \( (P = P_E = k_0/L) \). The general graphical representation of the \( H(\Delta) \) relationship is showed in Figure 2.3.1.2.

The analysis of the case \( p > 1 \), i.e. \( P > P_E \), is not interesting in the seismic situation, since buildings are designed with large safety factors against instability induced by only vertical loading. Therefore, only cases with \( p \ll 1 \) are discussed here, which is equivalent to consider only cases with \( k_{ht} > 0 \). The total lateral stiffness of the system when the base spring is in the plastic-hardening range \( (k_{ht}) \) may instead be positive or also negative (see the expression of \( k_{ht} \) given in Figure 2.3.1.2). In Figure 2.3.1.2, it has been assumed \( k_{ht} > 0 \), i.e. \( PL/k_h < 1 \). If this is not the case, the global softening, i.e. the descending branch of the \( H(\Delta) \) relationship, starts for \( \Delta = \Delta_y \) \( (k_{ht} < 0) \) (see Figure 2.3.1.3). The third branch of the \( H(\Delta) \) relationship has always a negative slope because of the type of internal restoring force characteristic (i.e. moment vs. rotation relationship of the base spring) taken into account \( (k_s < 0 \Rightarrow k_{st} < 0, \text{see the expression of } k_{st} \text{ given in Figure 2.3.1.2}) \). As shown in Figure 2.3.1.3, the descending third branch of the lateral force vs. lateral displacement relationship may be activated in the range \( H > 0 \) (full line), or may not be activated in this range (dashed line).
Figure 2.3.1.2. The monotonic lateral force vs. lateral displacement relationship in case of a tri-linear moment vs. rotation relationship of the base spring.

The intersection of the $H(\Delta)$ curve with the horizontal axis ($H = 0$) represents a very important point from the perspective of the lateral dynamic stability of the SDoF system, as it has been deeply discussed in the previous section.

As shown by Figure 2.3.1.2, or also by Figure 2.3.1.3, the lateral strength is not a monotonic function of the lateral displacement, but a maximum value of
lateral strength is reached for a particular value of lateral displacement. This value of lateral displacement is called here the 'static stability' limit displacement (indicated by the symbol $\Delta_{ss}$), meaning that it divides $\Delta$ into stable and unstable regions for static response to lateral static loads. It has to be clearly highlighted that $\Delta_{ss}$ is, in general, different from $\Delta_s$ ($= \phi_s L$), the latter being the limiting value of the lateral displacement corresponding to local static instability, i.e. to the activation of the plastic-softening branch in the moment - rotation relationship of the base spring. In fact, if $k_{hl} > 0$ then $\Delta_s = \Delta_{ss}$ (Figure 2.3.1.2), while if $k_{hl} < 0$ then $\Delta_{ss} = \Delta_y < \Delta_s$ (Figure 2.3.1.3).

However, it is usual that the level of vertical loading is sufficiently small to allow the development of significant local plastic deformation before reaching the maximum lateral strength, and, therefore, $\Delta_s = \Delta_{gs}$.

It is well known that the achievement of the static stability limit displacement ($\Delta_{ss}$) during an earthquake does not mean the collapse of the structure, since dynamic equilibrium is possible also along the descending branch of the $H$–$\Delta$ relationship, thanks to inertia forces. This concept is well known and will be not further discussed here. However, it is deemed important to emphasise that the achievement of $\Delta_{ss}$ is to be considered with particular judgement (Gupta and Krawinkler 2000a), since it implies a decreasing of lateral strength for further increasing lateral displacements. As it has been anticipated with reference to the elastic – perfectly plastic hysteresis model, the zeroing of lateral strength ($H = 0$) corresponds to collapse (dynamic instability) under the earthquake.

The reduction of lateral strength occurring beyond the static stability limit displacement is generally due to both the destabilising effects related to vertical loads and the decrease of internal strength occurring for increasing plastic deformations. The latter is related to the type of $M(\phi)$ relationship. Consequently, another limit displacement exists, beyond which equilibrium is possible only under a stabilising lateral force, i.e. under a force opposite in sign to the displacement. This limit displacement ($\Delta_c$), corresponding to zero lateral strength, is referred to as the 'collapse lateral displacement' (or as the 'critical lateral displacement'), as also already said in the previous section.

In order to obtain an analytical expression of $\Delta_c$ in the case of a tri-linear moment vs. rotation relationship, it is convenient to equate to zero the lateral strength $H$ given by formulae 2.3.1.6 and 2.3.1.7. Considering the two
different cases $\Delta_c > \Delta_s$ (see Figure 2.3.1.2) and $\Delta_c < \Delta_s$ (see Figure 2.3.1.3), we obtain the following two alternative equations determining $\Delta_c$:

from eq. (2.3.1.7) ($\Delta_c > \Delta_s$):

$$
\left(\frac{k_0 - k_h}{L^2}\right)\Delta_y + \left(\frac{k_h - k_s}{L^2}\right)\Delta_s + \frac{k_s}{L^2}\left(1 - \frac{PL}{k_s}\right)\Delta_{c,1} = 0
$$

(2.3.1.8)

from eq. (2.3.1.6) ($\Delta_c < \Delta_s$)

$$
\left(\frac{k_0 - k_h}{L^2}\right)\Delta_y + \frac{k_h}{L^2}\left(1 - \frac{PL}{k_h}\right)\Delta_{c,2} = 0
$$

(2.3.1.9)

From equation (2.3.1.8) and equation (2.3.1.9), the following expression for the critical value of the lateral displacement can be derived:

$$
\Delta_c = \min\{\Delta_{c,1} ; \Delta_{c,2}\}
$$

$$
\Delta_{c,1} = \left(\frac{k_0 - k_h}{PL - k_s}\right)\Delta_y + \left(\frac{k_h - k_s}{PL - k_s}\right)\Delta_s ; \Delta_{c,2} = \left(\frac{k_0 - k_h}{PL - k_h}\right)\Delta_y
$$

(2.3.1.10)

Introducing the displacement ductility $\mu = \Delta/\Delta_y$ we can define the ductility capacity as follows:

$$
\mu_c = \min\{\mu_{c,1} ; \mu_{c,2}\}
$$

$$
\mu_{c,1} = \frac{\Delta_{c,1}}{\Delta_y} = \frac{1 - k_h/k_0}{PL/k_0 - k_s/k_0} + \frac{k_h/k_0 - k_s/k_0}{PL/k_0 - k_s/k_0} \Delta_s = \frac{1 - h}{p - s} + \frac{h - s}{p - s} \mu_s
$$

(2.3.1.11)

$$
\mu_{c,2} = \frac{\Delta_{c,2}}{\Delta_y} = \frac{1 - k_h/k_0}{PL/k_0 - k_h/k_0} = \frac{1 - h}{p - h}
$$
where the following additional notation has been introduced: \( h = \frac{k_h}{k_0} \) and \( s = \frac{k_s}{k_0} \).

From a physical point of view, the collapse displacement computed as the minimum from equation (2.3.1.11) coincides with the critical value which could be directly computed by writing equilibrium in the displaced configuration under the vertical load only \((H = 0)\), as shown in Figure 2.3.1.4.

![Figure 2.3.1.4. Computation of the 'collapse displacement' by writing equilibrium in the displaced configuration under vertical loads only.](image)

Previous discussion about the evaluation of the ductility capacity is susceptible of a simple geometrical representation in the non-dimensional plane \( M/M_y - \mu \). In fact, the non-dimensional destabilising external moment is simply given by:

\[
\frac{P\Delta}{M_y} = \frac{P\Delta}{k_0\phi_y} = \frac{PL}{k_0} \Delta_y = p\mu
\]

(2.3.1.12)

that means it is a linear function of ductility \((\mu)\) for a given level of vertical loading \((\text{fixed} \, p)\).

The limit equilibrium equation illustrated in Figure 2.3.1.4 is thus rewritten, in a non-dimensional form, as follows:

\[
\frac{P\Delta_c}{M_y} = \frac{M_c}{M_y} \iff p\mu_c = \eta_c
\]

(2.3.1.13)
being, by definition, $\eta_c = M_c / M_y$.

Therefore, the ultimate ductility $\mu_c$ is obtained as the intersection of the straight line representing the normalised destabilising external moment with the tri-linear curve representing the stabilising internal moment, both taken as functions of the imposed displacement ductility. Figure 2.3.1.5 shows the two different cases $\Delta_c > \Delta_s$ ($\Leftrightarrow \mu_c > \mu_s$) and $\Delta_c < \Delta_s$ ($\Leftrightarrow \mu_c < \mu_s$): in the former case the intersection between the stabilising and destabilising curves occurs on the local softening branch, while in the latter case the intersection is on the local hardening branch. The transition from the case $\Delta_c > \Delta_s$ to the case $\Delta_c < \Delta_s$ occurs when the intersection of the straight line representative of destabilising moments with the moment-rotation curve of the base spring is just up to the point of maximum moment $M_s$ (Figure 2.3.1.6).

Figure 2.3.1.5. Graphical determination of the ductility capacity $\mu_c$. 
Figure 2.3.1.6. The limit situation where $\Delta_c = \Delta_s$ or, equivalently, $\mu_c = \mu_s$.

Assuming as main variable the parameter $\mu_s$ (i.e. the local softening ductility), a limiting value $\mu_s^*$ may be introduced. It characterises the transition from the case $\mu_c < \mu_s$ (intersection of the straight line representing destabilising external moments with the tri-linear curve representing internal resistance is on the local hardening branch) to the case $\mu_c > \mu_s$ (intersection of the straight line representing destabilising external moments with the tri-linear curve representing internal resistance is on the local softening branch). As can be seen in Figure 2.3.1.7, if $\mu_s > \mu_s^*$ then $\mu_s > \mu_c$, otherwise if $\mu_s < \mu_s^*$ then $\mu_s < \mu_c$. In other words, the limit value $\mu_s^*$ of $\mu_s$ split the ductility $\mu$ in regions of low local ductility ($\mu_s < \mu_s^*$) and high local ductility ($\mu_s > \mu_s^*$). In fact, when $\mu_s < \mu_s^*$ the local static stability limit state anticipates the global dynamic stability limit state ($\mu_s < \mu_c$). On the contrary, when $\mu_s > \mu_s^*$ the global dynamic stability limit state is achieved before the full exploitation of the stable part of local ductility ($\mu_c < \mu_s$).

The analytical expression of the limit value $\mu_s^*$ of the local softening ductility $\mu_s$ may be simply derived by writing the following equilibrium condition:

$$P\Delta_s = M_s \iff p\mu_s^* = 1 + h\left(\mu_s^* - 1\right)$$

$$\mu_s^* = \frac{1-h}{p-h}$$

(2.3.1.14)
It is interesting to notice that the expression of $\mu_s^*$ given by formula (2.3.1.14) coincides exactly with $\mu_{c,2}$, as defined by formula (2.3.1.11). This is not surprising, since the inequality $\mu_s > \mu_s^*$ imply that the critical ductility $\mu_c$ must be independent on both $\mu_s$ and $s$. Therefore, we can write:

$$
\mu_c = \mu_{c,1} \quad \text{if} \quad \mu_s \leq \mu_{c,2} \\
\mu_c = \mu_{c,2} \quad \text{if} \quad \mu_s \geq \mu_{c,2}
$$

(2.3.1.15)

Once fixed the parameters $h$, $s$, and $p$ (level of hardening, level of softening, and level of vertical loading, respectively) $\mu_{c,1}$ is a linear function of $\mu_s$ while $\mu_{c,2}$ is a constant (as shown by their expressions given in (2.3.1.11)). Thus, the $\mu_c$ vs. $\mu_s$ relationship looks like the qualitative picture of Figure 2.3.1.8.
Figure 2.3.1.8. A qualitative picture of the $\mu_c$ vs. $\mu_s$ relationship.

It is also interesting to evaluate the collapse lateral displacement in the case of an ideal elastic – perfectly plastic (EPP) type of behaviour for the base spring ($h = s = 0$ and $\mu_s$ undefined), which is a very commonly adopted model for representing plasticity. As already shown, the simpler and more meaningful way for computing the critical lateral displacement is to write the equilibrium condition under the vertical loads only, taking into account the second-order destabilising gravity-induced moments. In the case of an elastic – perfectly plastic moment vs. rotation response of the base spring, the resisting internal strength in the plastic range is constant and equal to the yield strength $M_y$. So, from the limit equilibrium equation, we can compute the following expression of the ductility capacity $\mu_c$:

$$ P\Delta_c = M_y \iff p\bar{\mu}_c = 1 $$

$$ \bar{\mu}_c = \frac{1}{p} \quad (2.3.1.16) $$

Obviously, in the case of an elastic – perfectly plastic type of moment vs. rotation relationship, the collapse ductility $\mu_c$ depends only on the level of vertical loading $p$, as also shown by Figure 2.3.1.9.
A frequently adopted parameter for measuring the local ductility capacity is that defined by the intersection of the constant yield moment line with the actual moment – rotation curve, as shown in Figure 2.3.1.10. This value of the local ductility is indicated herein by the symbol $\mu'_s$. The parameter $\mu'_s$ is very useful in the plastic design of structures under static and monotonically increasing external loads. In fact, when the ductility demand $\mu$ is less than $\mu'_s$, the actual available internal strength is greater than the yield moment $M_y$. This implies that the collapse load computed using the static or the kinematic theorem of plastic design of structures (i.e. using the elastic – perfectly plastic approximation for the local response in the plastic range) is reliable and safe, the actual collapse load being larger than that computed. It is indeed necessary, with this type of approach, to check that those components theoretically remaining elastic are actually able to carry out the hardened forces transmitted by plastic components, without yielding and thus changing the plastic mechanism type.

Figure 2.3.1.9. Graphical definition of the critical ductility $\mu_c$ in case of an elastic – perfectly plastic internal restoring force characteristic.

![Graphical definition of the critical ductility $\mu_c$ in case of an elastic – perfectly plastic internal restoring force characteristic.](image-url)
The mathematical expression of $\mu'_s$ may be simply found as a function of $h, s$ and $\mu_c$:

$$\mu'_s = \mu_s + \left[ \left( \frac{M_s}{M_y} - 1 \right) \right] = \mu_s + \frac{h(\mu_s - 1)}{|s|} \quad (2.3.1.17)$$

The assumption of an elastic – perfectly plastic approximation in the computation of the collapse displacement ductility $\mu_c$ could be safe or unsafe, depending on the ratio between $\mu'_s$ and $\mu_c$. In fact, if $\mu'_s < \mu_c$ then $\mu_c < \mu_c$ (see Figure 2.3.1.11a). On the contrary, if $\mu'_s > \mu_c$ then $\mu_c > \mu_c$ (see Figure 2.3.1.11b). Thus, the elastic – perfectly plastic approximation is safe in the case $\mu'_s > \mu_c$ and unsafe in the case $\mu'_s < \mu_c$. Remembering that $\mu_c = 1/p$ (see formula (2.3.1.16)), we can also say that the elastic – perfectly plastic approximation is safe if $p\mu'_s > 1$ and unsafe if $p\mu'_s < 1$. In other words, the elastic – perfectly plastic approximation is safe only in case of high levels of vertical loads (high values of $p$) and/or high levels of local ductility (high levels of $\mu'_s$). But, usually, ductility is limited and the vertical loads are sufficiently small to produce a significant impact of the type of plastic model on the computation of the critical ductility $\mu_c$. The influence of the local plastic behaviour modelling on the collapse ductility computation will be further discussed in the following paragraphs.
Figure 2.3.1.11. Situations where the elastic – perfectly plastic assumption is unsafe (a) and safe (b).

To better explain the influence of different analytical assumptions for the moment vs. rotation relationship on the computed value of the critical displacement ductility, some explicative diagrams are showed and discussed hereafter.

Figure 2.3.1.12 shows the effect of the slope of the local softening branch on the ductility capacity. Results are compared with the ones obtained assuming $\mu_c = \mu_s$ (dashed line). It is shown that the ductility capacity $\mu_c$ is strongly affected by the local softening phenomenon. In particular, the effect
is larger for smaller values of $\mu_s$, i.e. when local softening appears soon after the first yielding. Looking, for example, at the case $\mu_s = 1$, it is possible to observe that the ductility capacity may vary from around 1 to around 5, when $s$ changes from around –1 to around –0.03. Instead, in the case $\mu_s = 5$ the ductility capacity is around 5 for every value of $s$. Moreover, it can be observed that, for any given value of $\mu_s$, the rate of change of $\mu_c$ with $s$ is decreasing for increasing absolute values of $s$ (i.e. for increasing slopes of the softening branch). The latter observation implies that it is of lesser importance to accurately evaluate the slope of the softening branch when high values of $s$ are of concern. If we consider local softening as the result of local buckling in I-shaped sections of steel beam-columns, then the previous discussion implies that local buckling is strongly influencing low-ductility classes sections, as it is well expected. Besides, it can be said that for low ductility class sections the accurate evaluation of the slope of the post-buckling range is of lesser importance. In this latter case, it can be safely assumed $\mu_c = \mu_s$, neglecting the residual lateral displacement ductility capacity due to the post-buckling plastic reserves. But, if the post-buckling slope can be limited to small values (say less than 0.5) then the assumption $\mu_c = \mu_s$ may result in an excessive conservatism. Finally, we can observe that for $\mu_s > \mu_s^*$ the assumption $\mu_c = \mu_s$ may be strongly non-conservative.

![Figure 2.3.2.12. Influence of local softening phenomena on the global ductility capacity](image-url)
Chapter 2

Figure 2.3.1.13 discusses the adequacy of the assumption $\mu_c = \mu_s'$. Obviously, in this case we may obtain solutions much more unsafe than those obtained assuming $\mu_c = \mu_s$ (compare line (a) with line (1) and line (2)). Also, in cases where the assumption $\mu_c = \mu_s$ is conservative (compare lines (b) and (1)) the hypothesis $\mu_c = \mu_s'$ may be sometimes strongly unsafe.

Figure 2.3.1.14 shows the effect of the level of vertical loading $p$ on the collapse ductility. This effect is very large, leading to substantial variations of the lateral displacement capacity for not too large variations in the value of the parameter $p$. The achievement of dynamic instability is significantly affected by the level of gravitational loads, as well expected.

Figure 2.3.1.15 shows the effect of the level of hardening. As expected, increasing values of $h$ implies increasing values of the ductility capacity.

![Figure 2.3.1.13. Comparison of the assumption $\mu_c = \mu_s$ with the actual global ductility capacity.](image)
Figure 2.3.1.14. Influence of the level of gravitational loads on the global ductility capacity.

Figure 2.3.1.15. Influence of the level of hardening on the global ductility capacity.

Figure 2.3.1.16 shows the effect of the assumption of an elastic – perfectly plastic model for the moment vs. rotation response of the base spring. The assumption may lead to strongly non-conservative estimates of the ‘critical’
displacement ductility (compare line (a) and line (1)), or may sometimes implies very accurate estimate of \( \mu_c \) (compare lines (b) and (2)). But this is an exception, which occurs because of both the very high level of vertical loading and the very low level of hardening. Moreover, it is true only for \( \mu_s > \mu_s^* = \mu_{c,2} \). Therefore, it can be concluded that the a-priori assumption of an elastic–perfectly plastic type of internal restoring force characteristic as a realistic model of plasticity could be very dangerous when computing the dynamic stability limit ductility. Results concerning the global dynamic stability of structures and based on the elastic–perfectly plastic assumption for representing local plasticity are to be very careful judged, since they could be significantly unsafe. This conclusion, which has been derived here based on simple analytical relationships concerning the behaviour of a SDoF system, will be confirmed by numerical studies on MDoF structures, which will be shown in Chapter 5.

\[ \begin{align*}
(a): & \quad h=0.03 ; s=-0.5 ; p=0.1 \\
(b): & \quad h=0.03 ; s=-0.5 ; p=0.3 \\
\text{(1)}: & \quad \text{EPP model and } p=0.1 \\
\text{(2)}: & \quad \text{EPP model and } p=0.3
\end{align*} \]

**Figure 2.3.1.16. Inapplicability of the EPP model for computing the collapse ductility.**

### 2.3.2 The effect of cyclic strength degradation

In case of a generic lateral displacement history, the problem of defining the rule for the hysteresis behaviour of the base spring is to be faced. In particular, under generic deformation histories some deterioration phenomena may take
place, consisting in softening phenomena induced by plastic deformations cumulated in the base-spring. This type of strength degradation will be referred to as ‘cyclic’ strength degradation.

The paramount role of monotonic strength degradation in determining the lateral displacement capacity of a given structure has been shown in the previous section. It is apparent that the effect of strength degradation produced by the accumulation of plastic deformations is analogous to that of monotonic strength degradation.

In order to explain the role of cyclic strength degradation, let us consider first the case when the hysteresis behaviour of the base spring is cyclically stable. In this case, no local degradation phenomena take place and the lateral force vs. lateral displacement relationship of the SDoF system is also stable. This is shown, qualitatively, in Figure 2.3.2.1.

Figure 2.3.2.1. The lateral force vs. lateral displacement relationship of the SDoF system in the particular case of a stable hysteresis behaviour of the base spring.

Thus, stability of the hysteresis behaviour at the local level is transferred into stability at the global level. The system can be displaced laterally in the
range \([-\Delta_c, \Delta_c]\), without seismic stability problems. \(\Delta_c\) can still be defined as the critical lateral displacement of the system.

On the contrary, when ‘cyclic’ strength degradation takes place, any lateral displacement may become the critical one. In fact, a reduction of the internal resistance of the base spring anticipates the achievement of the lateral stability limit state, as shown in Figure 2.3.2.2 with reference to the simple case of a bi-linear non-hardening behaviour. As can be seen, the intersection of the internal (‘first-order’) moment resistance with the destabilising (‘second-order’) gravity-related moments curve could occur for a lateral displacement \((\Delta_{c,\text{res}})\) lesser than the limiting stability value relevant to a monotonic increase of lateral displacements \((\Delta_{c,\text{mon}})\). The symbol \(\Delta_{c,\text{mon}}\) indicates the lateral displacement capacity in case of monotonic loading, which as been introduced in the previous sections, while the symbol \(\Delta_{c,\text{res}}\) indicates the residual lateral displacement capacity. It is important to notice that the residual lateral displacement capacity is not unique (it is not a property of the system) but depends on the deformation history. In fact, different deformation histories induce different levels of cyclic strength degradation, and, therefore, different residual lateral displacement capacities.

**Figure 2.3.2.2. The effect of cyclic strength degradation on the lateral displacement capacity of SDoF systems.**
Evaluating the level of strength degradation actually induced by earthquakes is a problem of deformation demand evaluation. The cyclic strength degradation effects produce an interdependence of demand and capacity. On the contrary, as shown in the previous section, in the case of monotonic strength degradation, the lateral displacement capacity of the system could be defined based only on the monotonic force – displacement relationship, and is not affected by the cyclic deformation history. Then, in the latter case the system deformation capacity is not affected by the system deformation demand. On the contrary, in case of cyclic strength degradation, the lateral displacement capacity is not uniquely defined.

In any case, the approach developed throughout the current Chapter is applicable to every type of strength degradation, in order to individuate the achievement of the seismic stability limit state. Given the residual lateral displacement of the structure at the end of the earthquake, the system is seismically stable if this residual value is lesser than the residual lateral displacement capacity. The latter value can be computed according to the methodology shown in Figure 2.3.2.2, i.e. in a way analogous the that applied in the previous sections, simply substituting the initial monotonic lateral strength with its reduced value. It is apparent that the modelling of cyclic strength degradation is, in general, a necessary pre-requisite for analysing the system seismic stability.
Chapter 3  
Seismic Stability of MDoF Systems

3.1 INTRODUCTION

Among studies on dynamic stability of MDoF systems subjected to earthquakes, a relatively small number of attempts have been made in order to extend the main conclusion achieved by Jennings and Husid (1968). According to their intuition a SDoF system will go unstable during an earthquake when the deformation demand reaches a critical value corresponding to a zero lateral strength. This criterion has been deeply discussed in Chapter 2, where it has been shown, on a deductive basis, that a single-mass system displaced laterally up to a zero-strength lateral deformation is, in fact, in a dynamically unstable equilibrium condition. Consequently, it has been emphasised that the Jennings and Husid criterion is to be applied with reference to the residual lateral displacement of the system after the earthquake rather than to its maximum transient value. It has been emphasised that the critical lateral displacement of the mass can be computed using a simple 'static' approach, very similarly to what is usually done for checking the stability of elastic structures. Some simple analytical relationships have been derived, showing the formidable importance of strength degradation in determining the critical value of the ductility demand of the system.

A very notable work on the collapse safety evaluation of structures is that done by Akiyama (1985), especially for what concerns the practical
implications of his study in the design of earthquake-resistant structures in Japan. Akiyama applied the Jennings and Husid stability criterion to the case of 'shear-type' systems, identifying the collapse limit state of building frames characterised by rigid beams. The above Author extends the results obtained for shear type systems to the more general case of a frame with flexible beams using a procedure, not always immediate, which tries to reduce the real system to an equivalent simpler one. In any case, the Akiyama's approach is fundamentally based on energy concepts. In any step of his procedure, he emphasises the role of the energy-equilibrium equation as a synthetic tool for evaluating the structure seismic response.

Another very important contribution to the study of the stability of structures subjected to earthquakes has been given by Bernal (1992, 1998). He showed that a negative eigenvalue in the tangent stiffness matrix is a necessary but not sufficient condition for dynamic instability of the structure. Bernal has clearly emphasised the fundamental role of the type of collapse mechanism in the safety of the structure against dynamic instability. He has also obtained a number of numerical results aiming at the development of 'collapse' spectra, furnishing the minimum structural strength required to guarantee dynamic stability. The main limitation of Bernal's study derives from the use of non-degrading hysteresis models. As already emphasised, the prediction of the collapse safety of structures cannot always rely on the elastic-perfectly plastic hysteresis models, but the representation of mechanical degradation phenomena is generally needed. Moreover, the above Author, does not clarify how to identify the critical condition. Therefore, He does not furnish a criterion of seismic stability.

This brief overview of the existing knowledge on dynamic stability of structures subjected to earthquakes is terminated here emphasising that a large number of papers dealing with the problem of collapse safety evaluation of structures under earthquakes could be found in the technical literature. These papers treat the problem using a numerical approach and are, often, based on very refined hysteresis models. However, they do not furnish a clear methodology for computing safety against dynamic instability. It is contended that this limitation derives from a not complete understanding of the problem. In fact, the dynamic instability condition is not clearly identified, it being referred to simply as the condition leading to the divergence of the numerical
solution, i.e. simply applying the definition of dynamic instability. Notwithstanding, these papers are useful to approach the problem. For example, the paper by Challa and Hall (1994) and that by Hall (1998) are cited here.

The extension to MDoF structures of some conceptual findings of Chapter 2 is illustrated in the current Chapter. The basic philosophy that has led to the identification of the Seismic Stability Limit State of SDoF systems also applies to MDoF structures. The Seismic Stability Limit State is defined as that limiting damage condition where every internal lateral-strength source is completely devoted to make equilibrium to the gravitational loads. To be more precise, in the next paragraphs the structural collapse will be identified as that limit situation where the structure looses its capacity to sustain an indefinitely small increment of the lateral residual deformation produced by the earthquake.

The MDoF (Multi-Degree of Freedom) systems considered in the following are typical moment-resisting frame structures sometimes used as supporting skeleton of buildings. However, the concepts that will be derived are independent of the type of lateral-force-resisting system. Figure 3.1.1 shows a model for structural analysis of a three-storey two-bays moment-resisting frame. It is well known that, in the framework of seismic analysis of buildings, the lateral story displacements are fundamental kinematics parameters, the joint rotations being defined as a function of these displacements by means of the well-known static condensation of the stiffness matrix. Moreover, a sufficiently rigid concrete slab is assumed to exist at each floor level, so that the lateral displacement at each node on a given floor is uniform. Therefore, the deflected shape of the structure under lateral loads is identified by the vector $\Delta = (\Delta_1, \Delta_2, \ldots, \Delta_n)$. The lateral load distribution is contained in the vector $H = (H_1, H_2, \ldots, H_n)$. The sum of all gravitational loads acting on the i-th story is indicated by $G_i$. 
3.2 SEISMIC STABILITY BASED ON THE ELASTIC-PERFECTLY PLASTIC HYSTERESIS MODEL

Following an approach analogous to that used for studying SDoF systems in Chapter 2, a monotonic increase of lateral displacements will be first considered for discussion.

The extension of the concept of the critical displacement presented throughout Chapter 1 with reference to SDoF systems is quite easy in case of MDoF structures forming a plastic collapse mechanism, as shown in Figure 3.2.1. In this case, the deflected shape of the structure is characterised by one single kinematic parameter, which is the scale factor of the pre-fixed lateral plastic displacement pattern. Then, the lateral plastic deformation (displacement) capacity of the structure can be easily evaluated by writing a limit equilibrium equation. This equation indicates equilibrium under only gravitational loads in the laterally displaced configuration of the structure. Let us assume, for the sake of simplicity, that the effect of gravity loads on the

*Figure 3.1.1 Basic frame model parameters.*
initial frame configuration is an elastic state of internal forces without lateral displacements. Then the plastic lateral displacements will be related to rotations in the plastic hinges, whose plastic moments are equilibrated by the destabilising forces of gravity loads. This is shown with the help of Figure 3.2.1.

Using the virtual work principle, the equilibrium equation, under the destabilising lateral forces produced by gravitational loads, writes:

\[
8M_{pl} \dot{\phi} = \left( G_1 \dot{\Delta}_1 + G_2 \dot{\Delta}_2 + G_3 \dot{\Delta}_3 \right) \phi_c \quad \forall \phi
\]  

(3.2.1)

Noting that \( \dot{\Delta}_i = \dot{\phi}h_i \), where \( h_i \) is the height above the ground level of the i-th floor, gives us:

\[
8M_{pl} \dot{\phi} = \left( G_1 h_1 + G_2 h_2 + G_3 h_3 \right) \phi_c \phi \quad \forall \phi
\]  

(3.2.2)

Then, the critical rotation, for the considered plastic motion mechanism, can be derived as follows:

\[
\phi_c = \frac{8M_{pl}}{G_1 h_1 + G_2 h_2 + G_3 h_3}
\]  

(3.2.3)
The expression of $\varphi_c$ given by equation (3.2.3) is the plastic rotation capacity relevant to the pre-selected plastic motion mechanism. Obviously, a plastic deformation capacity can be defined for each plastic motion mechanism. Once, the plastic mechanism of the structure during the earthquake is known, the critical condition (divergence of lateral displacements) is attained when the plastic deformation demand equals the computed plastic deformation capacity. Currently, there are no methods to predict the plastic mechanism activated by a given ground motion for a given structure. Then, the methodology proposed should be considered as a performance evaluation tool rather than a predictive one. However, there is another important limitation of the approach based on the plastic motion mechanism. In fact, the structure could go unstable during the earthquake, even if no plastic motion mechanism has been formed. A clear example of such a situation is given in Figure 3.2.2, which is referred, for the sake of simplicity, to a single-mass system. In the case shown in Figure 3.2.2, the intersection of the straight line representing gravity-induced destabilising forces ($G\Delta/h$, $G$ being the gravity load, $\Delta$ the lateral displacement and $h$ the gravity load height on the ground level) with the first-order elasto-plastic lateral strength of the system occurs before the complete formation of a plastic motion mechanism. The ‘first-order’ lateral strength curve shown in Figure 3.2.2 could be referred, for example, to the system shown in Figure 3.2.3. It can be seen that, even the formation of only one plastic hinge could be in principle able to make the structure unstable, since it produces a lateral displacement.

Therefore, a general approach should be developed able to measure the frame damage also when a complete plastic motion mechanism has not been formed. Then, the Stability Limit State must be defined in the general case. It is deemed that, in case of elastic-perfectly plastic modelling of plastic hinges, a plastic motion mechanism is very likely to form before the structure becomes unstable. This opinion has been formed on the basis of some numerical studies on steel frames, whose vertical loading levels have been assigned based on loads acting on civil buildings. It is deemed that these loads are relatively small, so that a plastic motion mechanism will form before instability occurs. However, real structures are characterised by plastic hinges equipped with strength and stiffness degrading hysteresis behaviours.
Degradation could significantly impair the lateral strength of structures, thus producing the achievement of instability before the formation of a plastic motion mechanism. This reasoning, stimulated a deeper study of the problem, in order to propose a more general methodology, not relying upon the formation of a plastic motion mechanism. The formation of such a mechanism cannot be foreseen in general as well as the type of plastic motion in case the mechanism actually forms. This more general criterion of Seismic Stability is presented and discussed in the next paragraphs.

![Figure 3.2.2. Instability could occur before the formation of a complete plastic motion mechanism.](image)

![Figure 3.2.3. One plastic hinge can alone create destabilising effects.](image)

Equilibrium, in the displaced configuration of the MDoF system shown in Figure 3.1.1, writes:

\[
H = R_G(\Delta) - K_G\Delta \quad (3.2.4)
\]
The first term on the right hand side of equation (3.2.4) is the first-order internal restoring lateral load vector \( \mathbf{R} \). The subscript \( G \) emphasises that the \( \mathbf{R}(\Delta) \) relationship depends on the gravitational loads contemporary acting on the structure. It is worth noting that \( \mathbf{R}_G \) represents, in general, the elasto-plastic lateral strength of the frame, thus taking account of material non-linearity. Since we are considering a monotonic increase of lateral displacements, \( \mathbf{R} \) can be taken as a function of only \( \Delta \) (independent on \( \dot{\Delta} \) and other state variables).

The second term on the right hand side of equation (3.2.4) takes into account the effect of geometric non-linearity. This effect has been made linear in \( \Delta \), through the small displacements approximation, introducing the well-known ‘geometric stiffness’ matrix (Rutenberg 1982, Wilson & Habibullah 1987), which is taken as constant (i.e. independent on the lateral displacements). The subscript \( G \) emphasises that the geometric stiffness matrix depends essentially on the gravitational loads.

It is useful to briefly discuss the approximation contained in the assumption of a constant geometric stiffness matrix during the earthquake. As it is known, the generic term in the geometric stiffness matrix depends on the axial forces in the frame members. Owing to the dependence of the axial member forces on node displacements, the geometric stiffness matrix is rigorously a function of displacements itself. Then, some discussion is necessary to understand why it is possible to greatly simplify structural analysis assuming a constant geometric stiffness matrix. First of all, it is useful to notice that axial forces in beams, produced by lateral loads, are zero if a rigid floor diaphragm joint connection is assumed, as usual in seismic analysis of building structures. Thus, the source of geometric non-linearity is in the columns of the frame. It is well known that, for a generic beam-column member, geometric non-linearity could be considered as originated by two types of deformation: a rigid body rotation, producing what is sometimes called the 'P-\( \Delta \) effect', and the local flexural deformation of the member, producing what is sometimes called the 'P-\( \delta \) effect'. These two sources of geometric non-linearity are depicted in Figure 3.2.4. Let us discuss first the P-\( \Delta \) effect in the case of a building structure subjected to lateral (seismic) forces. Axial forces in columns of the frame depend on the lateral story displacements. Therefore, in the generic column of the frame, the P-\( \Delta \) effect is rigorously non-linear in \( \Delta \).
However, in case of seismic actions, the total vertical load acting on each floor remains constant. Earthquake–induced lateral loads produce an increase of axial forces in some columns and a decrease in other columns, but the mean value of the axial force in the columns sustaining a given floor remains constant. Thus, as noted by Wilson & Habibullah (1987), the P-∆ effect at the story level is unaffected by the lateral story displacements. It can be related only to the total vertical load acting on each floor and to the displacement of the floor itself relative to the adjacent lower one. Being the total vertical load on a given floor constant, the P-∆ effect at the story level could be treated as linear in $\Delta$, and a constant (independent on story displacements) geometric stiffness matrix could be assumed, at least for what concern the columns chord rotation effect. The discussion of the influence of the local P-δ effect on the story geometric stiffness matrix $K_G$ is somewhat more articulated. In fact, the use of a constant geometric stiffness matrix at the story level is allowed if variations due to local flexural deformations in some columns are compensated by opposite variations in other columns. This is true in the particular case of a shear-type frame, i.e. of a frame having rigid beams. In this case, under a given lateral story displacement distribution, the flexural deformation of all the columns at a given floor is the same (Figure 3.2.5). Thus, at least in this particular case, the variation of the story geometric stiffness matrix with lateral displacements is zero, the local flexural deformation producing a change in the member internal forces which is absorbed by the rigid beams. In the more general case of flexible beams, there will be an influence of the local P-δ effect on the global story geometric stiffness matrix and the problem should be treated as fully non-linear. However, it has been demonstrated that the P-∆ effect is generally dominant on the P-δ effect, when both of them are present in a beam-column member (Teh 2001). Thus, the geometric stiffness matrix can be considered approximately independent on lateral displacements induced by lateral loads and the geometric stiffness matrix computed using only gravitational loads can be used and taken constant throughout the earthquake-induced lateral deformation history.
Figure 3.2.4. $P$-$\Delta$ and $P$-$\delta$ effects.

Flexural deformations are the same for all the columns at a given floor. Since the resultant of axial forces in columns is constant during the earthquake, the variation of the lateral stiffness of the frame, owing to local $P$-$\delta$ effects during the earthquake, is zero.

Figure 3.2.5. Geometric non-linearity for 'shear-type' systems.
Now, go back to the equilibrium equation (3.2.4). The term $H$ on the left-hand side of equation (3.2.4) symbolises an external lateral load vector. The expression of $H$ given by equation 3.2.1 is defined here as the ‘restoring (lateral) load vector’, in the same manner as done for SDoF systems (see Chapter 2). The lateral load vector symbolised by $R$, which has been previously indicated as the first-order restoring lateral-force vector could be also named the 'internal restoring (lateral) load vector', in order to emphasise the difference with $H$. The former represents the first-order lateral strength, and is therefore unaffected by the P-$\Delta$ effect, whilst the latter represents lateral strength as affected by geometric non-linearity.

The system mechanical behaviour strongly depends upon the $R_G(\Delta)$ relationship, which in turn depends on the mechanical behaviour of each frame member and on connections between them. When the local response of frame components (members and joints) is well into the inelastic range, the $R_G(\Delta)$ relationship is strongly non-linear. Besides, the vector $R_G$ is the result of a complex interaction of every frame component. Therefore, even in the simplified case of a monotonically increasing pattern of lateral displacements, it is difficult to give a general analytical expression to this relationship. It depends both on the geometric compatibility conditions among members, given by the type of connection, and the type of mechanical behaviour of the single component (which is, by itself, a very complex phenomenon when large inelastic deformations are involved). Notwithstanding, some general considerations can be made.

When the response of any frame component is elastic, then the $R_G(\Delta)$ relationship is linear and independent on the gravitational load. In this case, the equilibrium condition under the lateral load $H$, can be written, as is well-known, in the following simplified form:

$$H= K\Delta - K_G \Delta = (K - K_G) \Delta$$

(3.2.5)

where $K$ is the linear (first-order) stiffness matrix of the frame. In this particular case, the (elastic) stability limit state is achieved for a level of vertical load given by the following condition:

$$H= 0 \iff (K - K_G) \Delta = 0 \iff \det(K - K_G) = 0$$

(3.2.6)
Instability of elastic structures, according to the Euler buckling load approach synthesised by equation (3.2.6), is very often referred to as 'static instability'. The term gives rise to some confusion about the physical meaning of the problem. In fact, elastic stability is, as it is well known, a genuine dynamic problem. The term 'static' refers to a simplified method of solving the problem (through a static approach), not to its intrinsic nature. It will be shown that also the elasto-plastic seismic stability problem can be solved with an analogous 'static' approach, by substituting the elastic internal strength \((K \Delta)\) with the actual elasto-plastic one \((R_G)\) and taking into consideration of the vertical loads acting on the structure.

In the seismic case, the safety factor against the elastic stability limit state is always very large (around 10 for well-engineered steel structures). Therefore, the actual collapse multiplier of gravitational loads will be less than the elastic critical one and its evaluation implies consideration of the non-linear material behaviour. However, also in this case, the safety factor for the building is always sufficiently high, because the structure stress state under vertical loads has been surely checked. It has been guaranteed that under the service value of the vertical load the structure will be in the elastic range of behaviour.

To explain the method of identification of the dynamic stability limit state of elasto-plastic structures subjected to earthquakes, let us consider first a system whose components are characterised by a perfectly plastic behaviour, without degradation of mechanical properties. Strength degradation effects will be discussed in Sections 3.3 and 3.4.

When a sufficiently strong seismic action will come, it will produce plastic deformations of the structure. Assume that the structure will not collapse under this earthquake action. This means that after the earthquake the structure is still able to guarantee static equilibrium under vertical loads. The equilibrium condition, after the earthquake \((H = 0)\), writes:

\[
R_G(\Delta_r) = K_G \Delta_r \tag{3.2.7}
\]

where \(\Delta_r\) is the vector describing the final residual lateral story displacements after the earthquake. \(\Delta_r\) is the sum of an elastic and a plastic (permanent) part:

\[
\Delta_r = \Delta_{re} + \Delta_{rp} \tag{3.2.8}
\]
By definition, the elastic part of the residual story displacements disappears if gravitational loads are removed and only the plastic part will remain to define a permanently deformed structure. The permanent residual story displacements are, therefore, an auto-equilibrated distortion of the structure and the following relationship holds true:

\[ \mathbf{R}_G(\Delta_{rp}) = \mathbf{0} \]  

(3.2.9)

Interpreting the plastic residual story displacements as an imposed distortion of an elastic structure, and taking into account relationship (3.2.9), the left-hand side of equation (3.2.7) can be rewritten in a simplified form:

\[ \mathbf{R}_G(\Delta_{re} + \Delta_{rp}) = \mathbf{R}_G(\Delta_{re}) + \mathbf{R}_G(\Delta_{rp}) = \mathbf{R}_G(\Delta_{re}) \]  

(3.2.10)

Assuming that the residual stiffness of the structure is equal to the elastic initial one, the first order elasto-plastic strength required to make equilibrium to gravitational loads in the deformed configuration at the end of the earthquake, can be simply expressed through the first-order stiffness matrix:

\[ \mathbf{R}_G(\Delta_{re}) = \mathbf{K}\Delta_{re} \]  

(3.2.11)

Then, the equilibrium condition (3.2.7) is finally written as:

\[ \mathbf{K}\Delta_{re} = \mathbf{K}_G\Delta_r \]  

(3.2.12)

The right-hand side of equation (3.2.12) could be interpreted as an external lateral load vector, equivalent to the residual lateral displacements, applying a well-known technique within the 'second order' analysis of geometrically imperfect structures (notional load approach).

The elastic part of the residual story displacement vector \( \Delta_{re} \) in equation (3.2.12) is limited by its yielding value \( \Delta_y \). Then, the Stability Limit State is achieved when the structure is on the verge of yielding under the lateral loads \( (\mathbf{K}_G\Delta_r) \) equivalent to the residual story displacements \( (\Delta_r) \). This result is synthesised as follows:

\[ \mathbf{K}\Delta_{ry} = \mathbf{K}_G\Delta_c \rightarrow \text{structure at the Stability Limit State} \]  

(3.2.13)
where the vector $\Delta_c$ is the 'critical' (or 'collapse') lateral displacement vector. It is just defined by equation (3.2.13) as that vector of residual lateral displacements, generating fictitious gravity-related lateral forces which, if applied at the end of the earthquake on the structure, produce its first yielding.

It is useful to remark that the post-earthquake yielding strength of the structure ($K\Delta_{ry}$), under the equivalent lateral loads defined by the right-hand side of equation (3.2.12) ($K_G\Delta_c$), is not the yielding strength that would be computed on the initial, undamaged, structure under the same lateral loads. In fact, the post-earthquake ‘first-order’ yielding strength depends on the ‘first-order’ stress state produced by vertical loads contemporary acting on the structure and by the residual stress state produced by residual plastic deformations acting as imposed distortions after the earthquake. In order to better clarify this important concept, in Figure 3.2.6 the complete residual (post-earthquake) stress state in the structure (which is an elastic stress state as previously remarked) is interpreted as the superposition of three contributions:

1) a ‘first order’ stress state produced by gravity loads acting on the initial (undamaged) structure state;
2) a ‘first order’ stress state produced by lateral loads equivalent, through the geometric stiffness matrix, to the residual lateral displacements;
3) an auto-equilibrated stress state produced by residual plastic deformations of the structure acting as imposed distortions after the earthquake.

It is interesting to notice that, if at collapse (Seismic Stability Limit State) the structure has already formed a plastic motion mechanism, then its post-earthquake first order yielding strength coincides with its pre-earthquake first order collapse load. In other words, in the particular case when a sufficient number of plastic hinges as formed to produce a mechanism, the following relationship holds true:

$$K\Delta_{ry} = R_{ry} = R_{0,m}$$  \hspace{1cm} (3.2.14)

Equality (3.2.14) can be easily understood looking at Figure 3.2.4, which relates the ‘first order’ lateral load multiplier and the top story sway displacement (push-over curve).
The structure is in equilibrium under gravitational loads in the deformed configuration after the earthquake. The stress state in the structure in this residual equilibrium condition can be computed superimposing results of the three following simpler schemes.

Fictitious lateral forces, equivalent to the destabilising effect of the gravitational loads acting on the laterally displaced configuration of the structure, are applied on the initial (undamaged) structure.

The gravity load is applied on the initial (undamaged) structure.

Plastic deformations produced by the earthquake can be applied as imposed distortions of an elastic structure.

*Figure 3.2.6. Decomposition of the residual stress state in three contributions.*
Figure 3.2.7. Coincidence of the post-earthquake 'first order' yielding strength and pre-earthquake 'first order' static collapse load in case of formation of a plastic motion mechanism.

As shown in Chapter 2, the Seismic Stability Limit State is achieved when the structure is deformed laterally up to the point of intersection of the first order lateral strength curve of the structure and the straight line representing destabilising lateral forces produced by P-Δ effects. Therefore, as can be seen in Figure 3.2.7, if the above intersection occurs when the maximum ‘first order’ internal lateral strength has been achieved then the pre-earthquake ‘first order’ collapse load coincides with the ‘first order’ post-earthquake yielding strength. It is apparent that both the yielding and the collapse load multipliers are computed with reference to the lateral force pattern defined by the residual lateral displacement pattern. Then, in the particular case shown in Figure 3.2.7, instead of computing the post-earthquake yielding strength of the structure, the pre-earthquake collapse load multiplier can be evaluated. The latter computation is rather simple to do with current existing numerical codes, which very frequently do not allow the direct application of imposed distortions on the structure, but usually are able to make a push-over first-order analysis. On the other hand, if the available numerical code allows the application of distortions, then the computation of the first yielding strength of the structure does not require any further computation into the inelastic range of behaviour. Moreover, the method illustrated in Figure 3.2.6 is more general and has a direct clear physical meaning.
The right-hand side of equation (3.2.13) could be considered the seismic demand, while the left-hand side is the seismic capacity. Then, equation (3.2.13) states that collapse occurs when demand is equal to capacity. However, it should be noted that, actually, the seismic capacity is not uniquely defined, but it depends on the residual lateral displacement pattern, which correspondingly generates the residual gravity-related force pattern. At different patterns of residual lateral displacements correspond different structural capacities. For example, in case of formation of a plastic motion mechanism, at each of it could be associated a lateral displacement capacity, to be computed as shown in Figure 3.2.1 and by equation (3.2.1). This is a main difference between SDoF systems characterised by only one type of collapse mechanism and MDoF structures, where multiple collapse mechanisms could be activated. At the time, there is no way to predict, with sufficient confidence, the type of collapse mechanism of a given structure during a given earthquake. Then, there is no possibility to predict the lateral displacement capacity of the structure. The effects of strength degradation, which will be discussed in the next section, further complicate the problem. Therefore, at the current state of the knowledge, the achievement of the Seismic Stability Limit State can be identified only via numerical analyses. In other words, the criterion stated by relationship (3.2.13) can be applied as a checking method, rather than a prediction tool.

Another way for obtaining equation (3.2.13) is explained in the following, with the aim to better clarify the procedure proposed to identify the Seismic Stability Limit State of the structure.

Let us consider the whole loading process of the structure when it is subjected to an earthquake. It can be considered to be made of two phases: 1) in the first phase gravitational loads are applied to the structure; 2) in the second phase a strong earthquake shakes the structure. Let us assume that the structure has survived the earthquake, even if some plastic deformation was produced in it. At the end of the earthquake, there will be residual story displacements (\(\Delta\)). Let us consider now an ideal post-earthquake loading process, composed of other two phases: 1) gravitational loads are completely removed from the structure; 2) the same gravitational loads acting before the earthquake are re-applied to the structure. This ideal loading process is fundamental to judge if the structure is stable. When unloading the structure, it
is apparent that only the elastic part of the residual story displacements ($\Delta_r = \Delta_r - \Delta_{rp}$) will be recovered and the permanent story displacements ($\Delta_{rp}$) will become apparent. Then, re-applying gravity loads (phase two of the ideal process), the structure will surely respond within the elastic range. Thus, taking into account the P-\(\Delta\) effect also during the re-loading elastic phase, the equilibrium condition could be written in the following form:

\[
(K - K_G)(\Delta_r - \Delta_{rp}) = K_G \Delta_{rp}
\]  
(3.2.15)

where the symbols have the meanings already defined. Eliminating the term $K_G \Delta_{rp}$, which appears on both sides of equation (3.2.15), the equilibrium condition is rewritten as follows:

\[
K(\Delta_r - \Delta_{rp}) - K_G \Delta_r = 0
\]  
(3.2.16)

which, remembering that $\Delta_r - \Delta_{rp} = \Delta_{re}$, is exactly coincident with equation (3.2.12). The structure is on the verge of collapsing, i.e. it is at the Seismic Stability Limit State, when it is at the first-order yielding limit state under the combined action of gravity loads and the fictitious lateral loads contained in the vector $K_G \Delta_r$. Thus, the simplest structural analysis tool can be used for solving a problem apparently much more complicated: a first-order elastic static analysis is sufficient to judge the post-earthquake stability of a structure. Really, it has to be considered that, for applying the above approach, the earthquake-induced permanent story displacements must be known. Therefore, at the time, the proposed methodology is a seismic performance evaluation tool rather than a prediction one.

If a non-degrading hysteresis behaviour is assumed for plastic zones, then the earthquake-induced permanent story displacements ($\Delta_{rp}$) fully represents damage to the structure. But, if plastic hinges are characterised by strength degradation, it is needed to adequately represent this phenomenon in the seismic stability evaluation methodology. It is apparent that, generally speaking, the effect of local strength degradation could be determinant on the seismic stability of the whole structure, as already discussed deeply in Chapter 2 with reference to SDoF systems. In fact, the global stability limit state could be achieved in a deformation range where significant local strength degradation has occurred. This is the case of civil buildings, which will be
shown in Chapter 5 through numerical analyses. Thus, in the next section, the concept of seismic stability represented by the relationship (3.2.13) will be extended to the case of strength degrading hysteresis behaviours.

3.3 SEISMIC STABILITY BASED ON DEGRADING HYSTERESIS MODELS

3.3.1 The effect of monotonic strength degradation

The seismic stability criterion introduced in the previous section (see relationship (3.2.13)) allows a very easy implementation of the effects of strength degradation.

In case of strength-degrading hysteresis behaviours of plastic hinges, at the end of the earthquake the yielding strength of each structural component, engaged in the plastic range of response during the previous earthquake action, will be reduced to some extent. Then, it is sufficient to substitute the left-hand side of the relationship (3.2.13), which is the pre-earthquake yielding lateral displacement vector associated to the lateral force pattern reported on the right-hand side of the same relationship, with the analogous quantity reduced to take account of residual strength of structural components:

\[ K\Delta_{r,\text{red}} = K_G\Delta_c \rightarrow \text{structure at the Stability Limit State} \] (3.3.1.1)

Analogously, in case of formation of a plastic motion mechanism, the 'first order' static collapse load multiplier, relevant to the lateral force pattern described by the vector \( K_G\Delta_c \), will be evaluated using the reduced values of the yielding strength of structural components at the end of the earthquake.

From equation (3.3.1.1) it is apparent that simulating strength degradation could have a formidable importance in evaluating the Structural Stability Limit State under seismic actions, since the structure capacity (left-hand side of equation 3.3.1.1) could be significantly reduced.

It is useful to remark that, actually, strength degradation modifies also the term \( \Delta_c \), which generates the destabilising gravity-related fictitious lateral forces at the end of the earthquake. There could be, in general, both changes in the values of each story lateral displacement and modifications of the whole...
pattern of residual story displacements. In other words, strength degradation could change, in principle, also the mode of the residual deformation of the structure. This is an additional difficulty in the prediction of the Structural Stability Limit State under earthquake actions, since strength degradation could change also the collapse mode of the structure. Unfortunately, the evolution of strength degradation is itself strictly related to deformation demand. In conclusion, demand and capacity are strictly inter-related at the Structural Stability performance level, thus making its prediction quite difficult without using a numerical approach.

However, if the structure deformation mode could be reasonably predicted then demand and capacity become independent each other, also in presence of monotonic strength degradation. In fact, in case of a predictable deformation mode, the critical multiplier of lateral displacement can be a-priori computed. For example, in case of formation of a plastic motion mechanism, the lateral plastic displacement capacity can be evaluated with the approach shown by Figure 3.2.1. The equilibrium equation (3.2.1) must be rewritten, in this case, substituting to the initial plastic strength of beams and columns, a reduced value, taking account of strength degradation. The reduced value will be, in general, a function of the maximum displacement. Then, the equilibrium equation (3.2.1) becomes a non-linear equation, which could be solved by iteration (see also Figure 2.3.1.4 of Chapter 2).

### 3.3.2 The effect of cyclic strength degradation

Effects of strength degradation produced by the repetition of plastic deformations of given amplitude (cyclic strength degradation) are analogous to those produced by strength degradation controlled by the maximum deformation demand (monotonic strength degradation).

Mathematically, effects of cyclic strength degradation on the Seismic Stability Limit State can be taken into account using again equation 3.3.1.1. But, the yielding strength of the structural components engaged in the plastic range of deformation should be reduced not only as a function of the maximum deformation demand but also in relation with some hysteresis behaviour parameters (for example, the plastic dissipated energy or, equivalently, the cumulated plastic deformation demand). Therefore, cyclic strength degradation gives rise to a correlation between demand and capacity.
which cannot be eliminated, never in the case of formation of a plastic motion collapse mechanism. It is necessary to predict the whole deformation history (or, at lest, to predict deformation history-related hysteresis parameters) to make an estimate of strength degradation. Then, at the moment, the numerical approach seems the only way to take into account this potentially very influent degradation phenomenon.
Chapter 4
Global Damage Evaluation

4.1 OVERVIEW OF EXISTING SEISMIC DAMAGE MODELS

It is quite difficult to trace an exhaustive historical overview of the different seismic damage models existing in the technical literature. In fact, first studies on the subject dates back to more than 30 years ago. Moreover, the history is made by many contributions, each of which should be cited.

Until now, many attempts have been made to define a seismic damage measure through the use of one single parameter. It must be recognised that this methodology is attractive because of its simplicity. However, the main limitation of existing approaches is the fully empirical nature of the indices proposed. In fact, a very careful engineering judgement has to be exercised in applying the current seismic damage models that are largely based on experimental data and on empirical coefficients. Thus the extension to situations different from those which have led to the calibration of the models requires caution.

In the following some of the more used parameters are reviewed. The list is perhaps not complete and the discussion is not exhaustive. In fact, the aim of this section is only to emphasise a basic limit, which is intrinsic to the general philosophy the existing seismic damage indices are based upon. A quite detailed overview of the problem of seismic damage evaluation can be found in Williams & Sexsmith (1995).
Seismic damage parameters are divided into two main groups: local (member level) and global (structure level) damage indices. Local damage indices are subdivided considering their ability to take into account either the effects of maximum deformations or the effects of cumulated deformation or both effects.

A list of local damage indices considering only the peak value of response parameters is the following:

- Ductility ratio: it is the ratio of the maximum deformation to the yield deformation.
- Inter-story drift: it is the ratio of displacement of one floor relative to the adjacent lower one and the story height.
- Slope ratio: it is the ratio between the tangent slope of the loading branch of the force displacement diagram and the slope of the unloading branch.
- Flexural damage ratio: it is the ratio of initial stiffness to the reduced secant stiffness at the maximum displacement.
- Maximum permanent drift: it is the residual (plastic) value of the inter-story drift after the earthquake.

Some local damage indices considering only cumulative damage effects are listed here:

- Normalised cumulative plastic deformation: it is defined as the sum of all plastic deformation excursions experienced by the structure.
- Normalised hysteretic energy: it is the ratio of the dissipated hysteretic energy to the product of the yield force and the ultimate deformation under monotonic loading conditions.

Some examples of local damage indices considering both the peak value of response parameters and the cumulative damage effects due to repetition of plastic deformations are:

- Park and Ang local damage index: it is an index obtained by the linear convex combination of the ductility ratio and the normalised hysteretic dissipated energy; the coefficients of the linear combination are empirically calibrated using experimental databases.
- Low-cycle fatigue index: it is an index based on a theory that tries to extend to the plastic range the fundamental concepts experimentally proved for the elastic fatigue problem.
Damage parameters are often further normalised so that to obtain a number ranging from 0 to 1. A normalised damage index equal to zero should correspond to no damage, a value equal to 1 should correspond to local failure. A main problem with the use of these damage indices is not only that they are calibrated against a limited database but also that the database is often referred to a specific limit state for the structural element under investigation. Thus, the damage index does not correlate well with damage corresponding to a different limit state (for example, a damage index calibrated on the basis of flexural limit states does not correlate well with a damage index related to a shear limit state). Therefore, any damage index should be calibrated using experimental results coming from different damage situations, so that to obtain a damage index for each type of limit state.

Global damage indices are obtained by combining the local values obtained through response analysis of the structure. The more frequently used combination is a weighed average of the local values, using the locally dissipated plastic energy as weighing factor. As highlighted by Ghobarah et al. (1999) this weighing scheme may lead to misleading results, attributing more damage to frames subjected to a lesser lateral force and with a lesser number of plastic hinges. Other weighing schemes have been proposed, basing on the use of the replacement cost of a damaged structural member and/or the relative importance of the member or substructure in maintaining the integrity of the structure. In any case, the global damage indices defined as combination of local values of selected damage parameters represent simply a numerical indicator of the damage intensity, without any physical meaning. Moreover, it is very difficult to select limit values of the global damage indices, which indicate the achievement of a given performance level.

DiPasquale & Cakmak (1988) proposed an interesting approach to the problem of obtaining information about global damage of the structure. They use the change in the fundamental period of vibration as a measure of the change in stiffness. They considered both a maximum value of damage, which is obtained by evaluating the fundamental period of vibration of an equivalent varying linear system during the time-history of the response (maximum softening) and a final value, which is obtained by means of the post-earthquake period of vibration (final softening). The final damage could also be measured in-situ after the earthquake. Obviously, this global softening
index cannot be used alone to give indication on the state of damage. In fact, a significant change of the fundamental period of vibration of a given structure occurs owing to non-structural damage (such as damage to infill walls). However, the basic idea of measuring the change of a structural parameter after the earthquake is very attractive because it is based on a clear physical meaning.

Ghobarah et al. (1999) also proposed a damage index based on the idea of measuring the state of damage by investigating on the change of the structure mechanical behaviour before and after the earthquake. The cited Authors proposed to perform two pushover analyses, one before the earthquake and the second after the earthquake (the latter being simulated by a given ground acceleration time-history). Thus, it is possible to evaluate, by comparison, the change of the mechanical behaviour of the structure. However, the Authors consider only a change of the initial stiffness of the structure, i.e. a change of the slope at the origin of the pushover curve.

The damage model proposed in the current study is basically related to the general idea that seismic damage is represented by the change of the structure mechanical properties caused by the earthquake. It is contended that measuring the seismic damage must be a way for measuring the seismic performance, i.e. for quantifying the structure ability to continue to serve, also after the earthquake, the function it was designed for, before the earthquake. When considering low-level structural performances (Life-Safety, Structural Stability, as defined in Chapter 1), the post-earthquake aspect, which is of major concern, is the evaluation of the building capacity to sustain vertical loads. At the 'Life Safety' structural performance level, the structure must be able to safely sustain vertical loads. At the 'Structural Stability' performance level, the structure is on the verge of collapsing for global instability. Thus, a well-defined and physically meaningful global damage index, must reflects this general philosophy of seismic performance evaluation. The global damage index that will be introduced in the following is developed looking at low-level structural performances, i.e. looking mainly to the building ability to safely sustain vertical loads after the earthquake. Taking in mind this general philosophy and according to concepts introduced previously, the earthquake-induced structural damage is considered to be the combination of two factors or damage types:
- A geometrical damage, which is represented by the residual plastic deformation of the structure produced by the earthquake.
- A mechanical damage, which is represented by the residual stiffness, residual strength and residual ductility of structural members.

If we are able to describe both two types of damage, then we are able to give a clear and rational picture of the structure physical state after the earthquake. But, it is necessary to introduce a quantitative description of damage, through the use of a synthetic 'damage index' able to consider both two types of damage in a rational way.

4.2 A ‘COLLAPSE-BASED’ GLOBAL DAMAGE INDEX

As discussed in the previous section, many damage parameters are currently used in earthquake engineering research studies, trying to empirically quantify the deteriorating effects of the earthquake action on structural members. The approach is very useful when the problem under investigation has to deal with structural members and modes of failure that are similar to those investigated experimentally. However, in situations different from those used for the calibration of the damage index it is quite impossible to obtain reliable information using the damage index itself.

It is deemed that, at the current state of the knowledge, it is impossible to analyse a structure in the inelastic range without using some information coming from experimental studies. Besides, it is contended that empirical information can be very useful if carefully and judgementally managed. However, it is also deemed that the current empirical efforts are not appropriately addressed. In fact, the real problem is to simulate, reliably and realistically, the hysteresis behaviour of structural elements. The use of a damage index (like the Park and Ang’s one) only add further empirical judgement to the problem of seismic damage evaluation without really solving it. This aspect is discussed in this section, where it is shown that, if we are able to accurately simulate the hysteresis response of structural elements, then we are able to meaningfully represent the seismic performance without the need to apply further empirical coefficients.

As briefly discussed at the end of the previous section, seismic damage is conceived to be composed of two damage forms or types:
- A *geometrical* damage, describing the change of the structure geometry produced by the earthquake.
- A *mechanical* damage, describing the reduction of the mechanical properties (stiffness, strength, ductility) of the structural elements.

Obviously, both damage forms are generally present on a given structure after a given earthquake, because of the degrading nature of the structural elements that dissipate the seismic input energy through plastic deformation. However, for the sake of clarity and simplicity, we will firstly examine the behaviour of a system equipped with non-degrading plastic zones.

Owing to plastic deformations and depending on both the seismic input and the structure mechanical characteristics, there will be a certain amount of residual permanent deformation of the structure after the earthquake. The permanent deformation may be directly used as a measure of damage, as already proposed by other Authors (see, for example, Kawashima *et al.*, 1998). It is apparent that excessive residual displacements may make a structure unusable, unsafe, and perhaps irreparable (MacRae & Kawashima, 1997). However, what really means ‘excessive’ is often a subjective matter. In case of evaluation of a non-structural performance level, the allowable residual displacements could be established on the basis of the type of non-structural element becoming unusable in presence of a large distortion of the initial building geometry. Thus, in this case, the limiting value of the residual deformation is dictated by consideration of non-structural systems. However, in case of low-level structural performances, such as the ‘Life-Safety’ or ‘Structural Stability’ levels, a rational criterion is needed to establish how much large could be a given residual deformation of the structure. In order to answer this question, it is essential to understand the consequences of having large residual displacements on a building. As deeply discussed in previous Chapters, inter-story drifts are the cause of P-∆ effects on a framed structure. P-∆ effects are, in case of non-degrading structural elements, the source of degradation at the global level. Residual story displacements, therefore, produce residual P-∆ effects, which are the source of risk for the structure post-earthquake safety. Owing to this residual P-∆ effect, the lateral strength of the structure is reduced, and its ability to survive earthquake aftershocks, or wind actions, may be seriously compromised. The reduction of the capacity of the structure to sustain further lateral loads can be measured through the
reduction of its lateral deformation capacity, as defined in Chapter 1. To better clarify this subject, consider first a SDoF system as shown in Figure 4.2.1, where the base spring is characterised by an elastic – perfectly plastic moment-rotation hysteresis behaviour. The P-Δ effect can be considered equivalent to the action of a lateral force proportional to the lateral displacement, as shown in the same Figure.

![Figure 4.2.1. A basic structural model for the development of a global damage index.](image)

As discussed in Chapter 1, the lateral deformation capacity is obtained imposing that the fictitious lateral load, equivalent to the P-Δ effect, is the yielding lateral load (see also Figure 2.3.1.4):

\[
\frac{P \Delta_c}{L} L = M_y
\]

(4.2.1)

where \(M_y\) is the yielding moment in the base spring. If the residual lateral displacement demand \(\Delta_r\) is equal to the lateral displacement capacity \(\Delta_c\) the structure is on the verge of collapsing: its residual lateral strength is zero due to excessive residual P-Δ effect. Therefore, the ratio between the residual lateral displacement and the displacement capacity is a good measure of the post-earthquake stability of the system:
\[ \text{Damage Index} = \frac{\Delta_r}{\Delta_c} \]  

(4.2.2)

The condition \( \Delta_r/\Delta_c = 1 \) identifies the 'Structural Stability Limit State'. When the ratio (4.2.2) is equal to zero damage is zero since there is no residual lateral displacement (remember that we are considering a non-degrading hysteresis behaviour for the base spring). The ratio \( \Delta_r/\Delta_c \) is then the measure of the geometrical damage, as previously defined.

Let us consider now strength-degrading hysteresis behaviours for the base spring. If \( M_{y,\text{res}} \) is an estimation of the residual plastic strength of the base spring after the earthquake, the residual 'collapse' lateral displacement is obtained as follows:

\[ \frac{P\Delta_c,\text{res}}{L} = M_{y,\text{res}} \]  

(4.2.3)

The displacement \( \Delta_c,\text{res} \), given by formula (4.2.3), represents the residual lateral displacement capacity of the structure after the earthquake. \( \Delta_c,\text{res} \) takes account of strength degradation through the use of a reduced value of plastic strength \( (M_{y,\text{res}}) \). When the residual lateral displacement demand is equal to the residual lateral displacement capacity \( (\Delta_r = \Delta_c,\text{res}) \) the structure is at the 'Structural Stability' performance level. Therefore, the damage measure parameter could be defined the following index:

\[ \text{Damage Index} = \frac{\Delta_r}{\Delta_c,\text{res}} \]  

(4.2.4)

The 'Damage Index' defined by formula (4.2.4) is a non-dimensional measure of the post-earthquake ability of the structure to sustain gravity loads (remember that \( \Delta_r = \Delta_c,\text{res} \) implies dynamic instability of the structure). However, it does not give a direct measure of the residual lateral strength of the structure. Since the engineer is more familiar with forces rather than displacements, it could be useful to develop an index expressing post-earthquake safety of the structure directly in terms of forces. Such an index is derived in the following.
The ‘force-based’ approach is explained with the help of Figure 4.2.2, where the ‘first-order’ lateral strength vs. lateral displacement relationship of a SDoF system is shown together with the gravity-induced destabilising lateral force produced by the P-\(\Delta\) effect. In Figure 4.2.2 the displacement capacity \(\Delta_c\) and the residual displacement \(\Delta_r\) are also shown. The gravity-induced lateral force, which is not an actual force acting on the structure but can be considered equivalent to the residual lateral displacement through the use of the geometric stiffness, is shown in Figure 4.2.2 by the symbol \(H_r\). Finally, in Figure 4.2.2 the residual yielding displacement (\(\Delta_{y,\text{res}}\)) and yielding force (\(H_{y,\text{res}}\)) are shown. The ratio between the residual yielding lateral force and the residual yielding lateral displacement is the residual lateral stiffness of the structure, which could be, in general lesser than the initial elastic stiffness due to degradation effects of plastic deformations. A lateral force acting on the structure after the earthquake will produce yielding when it reaches the residual yielding strength value (\(H_{y,\text{res}}\)) minus the gravity-induced residual lateral force (\(H_r\)). Then, the ratio \(H_r/H_{y,\text{res}}\) could be used for measuring the post-earthquake lateral strength of the structure in a normalised form:

\[
\text{Damage Index} = \frac{H_r}{H_{y,\text{res}}} \tag{4.2.5}
\]

The proposed Damage Index is a non-dimensional measure of the earthquake-induced global damage of the structure. It can be applied to both the case of non-degrading and degrading hysteresis modelling of plastic hinges. It is useful to notice that the index defined by formula (4.2.5) is also a ‘Stability Index’ (or, rather, an ‘Instability Index’). In fact, an increase of the Damage index means a reduction of the post-earthquake degree of stability of the structure. In the limit situation, when \(H_r = H_{y,\text{res}}\) (Damage Index = 1), the system is at its ‘Structural Stability Limit State’, i.e. it is on the verge of collapsing owing to excessive residual P-\(\Delta\) effect (large values of \(H_r\)) and/or excessive strength degradation (small values of \(H_{y,\text{res}}\)). Figure 4.2.3 highlight the limit situation where the Damage Index is equal to 1.
It is interesting to notice that the 'displacement-based' Damage Index (formula 4.2.2) coincides with the 'force-based' Damage Index (formula 4.2.5), in the particular case of an elastic – perfectly plastic hysteresis behaviour of the base spring. This is shown in Figure 4.2.4. This equivalence holds true also in case of strength-degrading hysteresis behaviours like those shown in Figure 4.2.5. The difference between the two indices is due only to a monotonic hardening and/or softening phenomenon, i.e. to a positive and/or negative stiffness in the plastic range of deformation.
The extension of the 'force-based' Damage Index defined by formula (4.2.5) to multi-degree of freedom systems is quite easy, if based on the idea of substituting residual displacements with the equivalent lateral forces. Under the fictitious lateral load distribution equivalent to the residual lateral story displacements of the structure, the safety factor against first yielding gives indication on the closeness of the Seismic Stability Limit State (see also equation (3.2.10)).
\[ \alpha_c K C \Delta_r = K \Delta_{ry, res} \] (4.2.6)

where \( \alpha_c \) is the 'collapse' multiplier of residual story displacements. The Damage Index, being defined as a quantity ranging from 0 (no damage) to 1 (collapse), is the reciprocal of the collapse multiplier:

\[ \text{Damage Index} = \frac{1}{\alpha_c} \] (4.2.7)

In other words, indicating by \( V_r \) the residual base shear, equivalent to the residual lateral displacements, and by \( V_{y, res} \) its post-earthquake 'first-order' yielding value, the Damage Index is given by

\[ \text{Damage Index} = \frac{V_r}{V_{y, res}} \] (4.2.8)

Both formula (4.2.5) and (4.2.8) clearly show that the proposed approach allows us to consider both the geometrical and the mechanical damage types. The numerator, through the concept of a residual base shear equivalent to residual lateral displacements, takes account of the geometrical damage while the denominator, which reduces when strength degradation takes place, considers the effects of mechanical damage. Actually, the effect of the mechanical damage is of double nature. In fact, degradation of mechanical properties has an effect also on the residual displacement demands, thus producing a modification of also the residual base shear. For example, elastic stiffness degradation has an effect on the numerator of formula (4.2.8).

In order to avoid confusion or misunderstanding, it is important to highlight that the yielding value of the residual base shear \( (V_{y, res}) \) must be computed taking account of the actual mechanical state of the structure after the earthquake. This is explained with the help of Figure 4.2.6, where the decomposition of the mechanical model of the structure into the summation of simpler structural schemes is shown. The Figure has been already reported in Chapter 3 (see Figure 3.2.3), but it is replicated here for convenience. As can be argued by the help of Figure 4.2.6, in the computation of the yielding value of the gravity-induced residual base shear, the stress state produced by both
gravity loads and plastic deformations, the latter acting after the earthquake as imposed distortions, must be considered. The yielding value of the generic internal force (for example the yielding moment in a plastic hinge) will take into account local degradation owing to excessive plastic deformations.

The effect of the residual stress state produced by residual plastic deformations, acting as imposed distortions of the structure, is shown with the help of a simple SDoF model, obtained by combining two springs in parallel. Both in Figure 4.2.7 and 4.2.8, one of the two springs has been equipped with a strength-degrading restoring force characteristic (component 1) while the other spring is assumed to be perfectly plastic (bi-linear without hardening, component 2). Figure 4.2.7 refers to the case where the Seismic Stability Limit State is achieved after the formation of a plastic motion collapse mechanism, i.e. after yielding of both two springs in this simple case. As it can be seen in Figure 4.2.7, in the residual state with a zero total lateral load \((H = H_{1r} + H_{2r} = 0)\), one spring is subjected to a negative internal force \((H_{1r} < 0)\), while the other is subjected to an equal but opposite force \((H_{2r} > 0)\). Thus, the post-earthquake yielding strength of the system does not coincide with the 'first-order' yielding strength that would be computed on the undeformed structure, simply reducing the yielding strength of component 1 \(H_{y,1r} < H_{1y}\). It is needed to consider the effect of the residual stresses in the structure, produced by the residual plastic deformation, in order to compute correctly its post-earthquake yielding strength under residual lateral loads. However, it is interesting to notice that the post-earthquake 'first-order' yielding strength can be also computed as the 'first-order' collapse load of the structure in its initial configuration (without distortions), simply using the residual plastic strength of each structural component. In fact, as can be seen in Figure 4.2.7, the post-earthquake yielding strength of the system is correctly given by \(H_{y,1r} = H_{1y,1r} + H_{2y}\), i.e. by the 'first-order' plastic collapse load of a strength-degraded structure \(H_{1y,1r}\) is used instead of the pre-earthquake, undamaged, value \(H_{1y}\). This procedure cannot be applied in the case where the Seismic Stability Limit State is achieved before the complete formation of the plastic motion mechanism. For example, in Figure 4.2.8, it has been hypothesised a different location of the residual displacement of the system such that the yielding of component 2 has not still occurred when the structure is at its Seismic Stability Limit State. As can be
seen in Figure 4.2.8, the 'first-order' plastic collapse load of the system, computed using the pre-earthquake structure geometry but the post-earthquake residual strength of each component, is larger than the actual post-earthquake 'first-order' yielding strength ($H_{1y,\text{res}} = \Delta H_{1y,\text{res}} + \Delta H_{2y} < H_{1y,\text{res}} + H_{2y}$).

Both the 'displacement-based' and the 'force-based' Damage Index previously introduced are zero if there are zero residual lateral displacements. In case of ductile structures, collapsing for excessive plastic lateral displacements, this is not a problem. But, in case of brittle structures, the proposed index are not able to capture earthquake-induced damages. In fact, a brittle structure is characterised by an elastic behaviour (no residual displacements) up to its collapse. In this particular case, the lateral collapse of the structure occurs in the form of an elastic instability, owing to excessive stiffness degradation. In order to consider also this limit case, the proposed approach could be simply improved by introducing geometric imperfections. Thus, a generalised form of the Damage Index is the following:

$$\text{Damage Index} = \frac{V_{y,0} - V_{y,\text{res}}}{V_{y,0} - V_r}$$  \hspace{1cm} (4.2.9)

where $V_{y,0}$ is the yielding lateral strength before the earthquake, $V_{y,\text{res}}$ is the yielding lateral strength after the earthquake and $V_r$ is the gravity-related residual base shear. $V_r$ is the sum of two contributions $V_{ri}$, which is the gravity-related residual base shear produced by geometric imperfections, and $V_{re}$, which is the earthquake-induced gravity-related base shear. When $V_r = V_{y,\text{res}}$ (Seismic Stability Limit State) the Damage Index is equal to 1, while it is equal to 0 if $V_{y,\text{res}} = V_{y,0}$ (no damage). Thus, also when $V_{re} = 0$ (no plastic residual deformations of the structure), the Damage Index could be not zero, because of strength and stiffness degradation.

Allowable values of the Damage Index could be fixed for each structural performance level. For example, at the Life Safety performance, the structure should be able to sustain vertical loads after the earthquake with an adequate safety margin against earthquake aftershocks or wind actions. Then the Damage Index, giving a measure of the reduction of lateral strength, should be limited to adequately small values.
The structure is in equilibrium under gravitational loads in the deformed configuration after the earthquake. The stress state in the structure in this residual equilibrium condition can be computed superimposing results of the three following simpler schemes.

Plastic deformations produced by the earthquake can be applied as imposed distortions of an elastic structure.

The gravitational load is applied on the initial (undamaged) structure.

Fictitious lateral forces, equivalent to the destabilising effect of the gravitational loads acting on the laterally displaced configuration of the structure, are applied on the initial structure configuration.

*Figure 4.2.6. Decomposition of the mechanical model of the structure into the summation of simpler structural schemes.*
Figure 4.2.7. Effect of residual stresses owing to residual plastic deformations in case of formation of a plastic motion mechanism prior to the achievement of the stability limit state.
\[ H = H_1 + H_2 \]

\[ H_{y,\text{res}} = \Delta H_{1y,\text{res}} + \Delta H_2 \]

\[ (H = H_{1r} + H_{2r} = 0) \]

Seismic Stability Limit State achieved prior to the formation of a plastic collapse mechanism (yielding of only component 1).

Figure 4.2.8. Effect of residual stresses owing to residual plastic deformations, in case of the achievement of the stability limit state prior to the formation of a plastic motion mechanism.
A different approach could be pursued to solve the problem of representing damage also for brittle structures. It is still a 'force-based' approach, but the force considered is not a lateral load but the gravity load. For the sake of simplicity, this approach will be discussed with reference to the elastic-perfectly-plastic hysteresis behaviour.

As stated by equation (4.2.1) the lateral displacement capacity of the structure is not affected by its elastic stiffness. In fact, the limit equilibrium, rewritten using explicitly the base spring rotation (see Figure 4.2.1), writes as follows:

$$P\varphi_c L = M_y$$  \hspace{1cm} (4.2.10)

where $\varphi_c$ is the 'collapse' rotation, $P$ is the gravity load, $L$ is the height of $P$ on the ground level, $M_y$ is the base spring plastic strength. Then, it appears clearly that the system lateral stiffness does not affect its deformation capacity ($\varphi_c$ is independent on $k$). However, it should be reminded that the achievement of the stability limit condition depends not only on capacity but also on demand. In fact, according to concepts previously introduced, a structure will survive the earthquake if and only if the residual rotation demand is smaller than the 'collapse' value established by formula (4.2.10):

$$\varphi_r < \varphi_c \hspace{1cm} ⇔ \hspace{1cm} \text{The structure survives the earthquake} \hspace{1cm} (4.2.11)$$

The lateral stiffness of the system directly affects the residual rotation demand ($\varphi_r$). Then, also stiffness degradation directly affects the term $\varphi_r$. This aspect differentiates stiffness degradation from strength degradation. In fact, the system lateral strength has a direct influence on both the deformation capacity and the deformation demand. The deleterious effect of a reduction in strength appears immediately looking at equation (4.2.10). On the contrary, it is not easy to highlight the deleterious effect of stiffness degradation, since the rotation demand is not a structural characteristics, but it depends also on the ground motion. The reduction in stiffness could have a beneficial effect on the required deformation, because of the shifting of the elastic vibration period of the structure into a more favourable zone of the demand spectra.

However, changing slightly the perspective, allows us to correctly interpret also the deleterious effect of stiffness degradation. In fact, the residual
deformation demand is the sum of an elastic part and a plastic (permanent) part, as shown by the following formula:

\[ \varphi_r = \varphi_{0r} + \Delta \varphi_r \]  

(4.2.12)

where \( \varphi_{0r} \) is the permanent part of the residual rotation and \( \Delta \varphi_r \) is its elastic increment produced by the destabilising effect of vertical loads. If the plastic part of the residual rotation is known, then the elastic part of the residual rotation could be determined from the non-linear equilibrium condition in the residual state of the structure after the earthquake:

\[ P(\varphi_{0r} + \Delta \varphi_r) = k \Delta \varphi_r \]  

(4.2.13)

The \( P \) vs. \( \varphi \) relationship, established by equation (4.2.13), is very well known in the field of elastic stability of structures, where the term \( \varphi_{0r} \) is usually introduced to represent geometrical imperfections. This relationship is qualitatively shown in Figure 2.3.3.1.

![Figure 4.2.9. A qualitative picture of the P vs. \( \varphi \) non-linear elastic equilibrium relationship (the small displacements approximation has been made).](image)

Equation 4.2.13 is valid up until the elastic part of the residual rotation is smaller than its yielding value. When the base spring plastic strength is reached or, equivalently, when the yielding rotation is achieved, the \( P \) vs. \( \varphi \) relationship changes. In fact, the equilibrium condition in the plastic range of
the base spring is expressed by equation 4.2.10. Its graphical representation is shown in Figure 4.2.10. The decreasing \( P \) vs. \( \phi \) relationship is usually called the 'plastic mechanism curve'. In the context of seismic stability of structures the plastic equilibrium curve represents, for any given level of gravity load (fixed \( P \)), the rotation capacity of the system (or collapse rotation, using terminology previously introduced). In order to avoid confusion with the generic value of vertical load, in Figure 4.2.10 the gravity load actually acting on the structure, during and immediately after the earthquake, has been indicated as \( P_r \).

![Figure 4.2.10. A qualitative picture of the \( P \) vs. \( \phi \) non-linear elasto-plastic equilibrium relationship.](image)

The elastic equilibrium curve is superimposed to the plastic one in Figure 4.2.11. The point of intersection of these two curves represents the well known static 'collapse load' of the structure. It is worth emphasising that, in the seismic case, this limit value of the vertical load is not fixed, but depends on the degree of damage produced by the earthquake.

In Figure 4.2.11, the residual total rotation demand (\( \phi_r \)) under the gravity load actually acting on the structure (\( P_r \)) is also shown, in the hypothesis that the structure has survived the earthquake. Looking at Figure 4.2.11, it clearly appears that the condition \( \phi_r = \phi_c \), which characterises the Seismic Stability Limit State, is equivalent to the condition \( P_r = P_c \). Thus, the qualitative description of the Seismic Stability Limit State given both in the Foreword and in Chapter 1 of this dissertation, is strictly respected. The Seismic
Stability Limit State is that limiting damage condition under which the structure loses its ability to sustain gravitational loads.

![Figure 4.2.11. A comparison of rotation demand ($\phi_r$) and rotation capacity ($\phi_c$).](image)

Before the earthquake, the structure is laterally stable because $P_r < P_E$ (initial geometrical imperfections are here neglected). After the earthquake, the plastic part of the residual deformation acts on the structure like a geometrical imperfection, reducing the collapse load to a value $P_c$ smaller than the theoretical Euler buckling load. When the reduction is so large that the residual collapse load is equal to the vertical load actually acting on the structure, the Seismic Stability Limit State is achieved.

Now, the effect of stiffness degradation can be more clearly discussed. This is made with the help of Figure 4.2.12, where the $P$ vs. $\varphi$ relationship of two different systems is shown. The two systems have been assigned the same plastic strength, but a different stiffness ($k_2 < k_1$). As previously discussed, the rotation capacity is unaffected by a change in the system stiffness, then the plastic equilibrium curve is the same for both two systems. On the contrary, the elastic equilibrium curve is different, the Euler buckling load being smaller for the system having the smaller stiffness ($P_{E2} < P_{E1}$). Consequently, the collapse load of the system more flexible is smaller than that of the system stiffer ($P_{c2} < P_{c1}$). Then, the deleterious effect of degradation of lateral stiffness becomes apparent. The vertical load acting on the structure will be closer to the collapse load for the system having the smaller stiffness. From a
dual perspective, the system having a smaller stiffness will have a larger residual total rotation, which will be closer to the rotation capacity.

\[ P \]

\[ P_{E1} \]

\[ P_{E2} \]

\[ P_r \]

\[ \phi_0 \]

\[ \phi_1 \]

\[ \phi_2 \]

\[ \phi_c \]

Figure 4.2.12. The effect of stiffness degradation on the achievement of the Seismic Stability Limit State.

A simple way of understanding the dangerous effect of stiffness degradation is to think over the limit case of a zero plastic residual rotation (brittle structures). In this case the structure could still collapse because of excessive stiffness degradation. This will occur if the lateral stiffness is so strongly reduced that the gravitational load acting on the structure becomes its Euler buckling load.

According to previous discussion about the deleterious effect of stiffness degradation, the Damage Index could be also defined in the following form:

\[
\text{Damage Index} = \frac{P_{E,0} - P_{c,r}}{P_{E,0} - P}
\]  

(4.2.14)

where \( P_{E,0} \) is the initial Euler buckling load, \( P_{c,r} \) is the residual lateral collapse load and \( P \) is the acting gravity load. As it can be seen, in absence of damage \( (P_{c,r} = P_{E,0}) \) the Damage Index will be zero, while it will be equal to 1 at collapse \( (P = P_{c,r}) \). The actual initial lateral collapse load (reduced because of initial imperfections) could be used instead of \( P_{E,0} \) in formula (4.2.14).
Chapter 5
Numerical Investigation on the Seismic Stability of MR Steel Frames

5.1 ANALYSED FRAMES

Several numerical results concerning the seismic stability of steel frames are presented in the next Sections. The frames analysed are part of the lateral resisting system of a five-storey civil building, whose plan is schematically illustrated in Figure 5.1.1.

![Schematic plan of the analysed steel building.](image)

*Figure 5.1.1. Schematic plan of the analysed steel building.*
The earthquake resistant frames are moment resisting (MR) and are located at the perimeter of the building. Thus, they will be indicated by the symbol PMR (perimeter moment resisting). They have been designed according to the European structural codes EC3 (CEN 1991) and EC8 (CEN 1994). In particular, a \( PGA \) (peak ground acceleration) equal to 0.35g, a soil type B (medium density sand) and a design strength reduction factor (behaviour factor) \( q_d = 6 \) have been assumed in the design phase.

Some discussion is necessary about the assumption used for satisfaction of the serviceability requirement. Eurocode 8 imposes that the maximum inter-story drift (relative displacement between the generic story and the adjacent lower one) under moderate earthquakes be limited to a value ranging from 0.004\( h \) to 0.006\( h \) (\( h \) = story height). The limiting value depends on whether the non-structural elements are rigidly connected to the main earthquake resistant structure or not. Eurocode 8 does not explicitly specify the earthquake intensity to be considered moderate, but requires the checking of the serviceability requirement starting from the displacements computed under the design forces, i.e. the seismic forces obtained by applying the design \( q \)-factor to the elastic response pseudo-acceleration spectrum. After a simple manipulation, the prescribed serviceability checking formula given by EC8 can be expressed in the following form:

\[
\frac{q}{\nu} \left( \frac{\Delta}{h} \right)_{e} \leq \left( \frac{\Delta}{h} \right)_{\text{lim}}
\]

(5.1.1)

where: \( q \) is the strength reduction factor allowing for consideration of the frame ductility, \( \nu \) is a factor accounting for the lower return period of moderate earthquakes with respect to the strong ones, \( (\Delta/h)_e \) is the inter-story drift angle computed within the elastic range under the design forces and, finally, \( (\Delta/h)_{\text{lim}} \) is the limiting value of the inter-story drift angle, allowable for control of non-structural damage. The meaning of formula (5.1.1) can be simply interpreted remembering the well-known 'ductility theory', which states that the elasto-plastic displacement demand is approximately equal to the elastic one of a system having the same elastic vibration properties as the actual system (see Figure 5.1.2).
If the 'equal displacements' approximation is considered to be valid, then the term $q(\Delta/h)_e$ represents the drift angle demand under the design earthquake, while $q(\Delta/h)_e/\nu$ represents the drift angle demand under moderate earthquakes. It is the same to say that EC8 implicitly assumes that the intensity of moderate earthquakes, under which the satisfaction of the serviceability requirement must be checked, is equal to $1/\nu$ times the intensity of the strong earthquake, under which the ultimate limit state must be controlled. The numerical value of $\nu$, adopted by EC8, depends on the importance category of the building. However, the range of variability of $\nu$ is very small, being $(2, 2.5)$. The assumption $\nu = 2$, which will be made in the following, does not affect the generality of the conclusion which will be hereafter drawn. For steel MR frames the ultimate limit state could be identified, in a simplified manner useful for discussion, with the achievement of an inter-story drift angle equal to 0.03 rad. In fact, neglecting the elastic part of the inter-story drift with respect to the total elasto-plastic value, the inter-story drift angle is very well correlated with the local plastic rotation demand in beams and/or columns. 0.03 rad is a value very often indicated as one that could be considered a reliable allowable limit of plastic rotation demand in order to avoid significant local strength degradation in the plastic hinge. Analogously, let us assume that the allowable value of the maximum
inter-story drift angle demand, relevant to the serviceability requirement, be equal to 0.005 rad, which is a mean value between 0.004 and 0.006 rad. Under these premises, with the help of Figure 5.1.3, we can demonstrate that the current codified design rules does not permit an optimum design of steel MR frames. In other words, it is contended that, following the codified EC8 design rules, it is impossible to satisfy, with the same level of confidence, both the serviceability and the ultimate limit state requirements. In order to demonstrate this assertion, in Figure 5.1.3 the base shear seismic coefficient $C$ (i.e. the ratio between the base shear and the seismic weight) is related to the inter-story drift angle $\Delta/h$. In the Figure, $C_{e,u}$ indicates the elastic base shear seismic coefficient relevant to the ultimate limit state of the structure, i.e. the seismic intensity which should produce the ultimate limit state. Instead, $C_{e,u}/q$ indicates the design base shear seismic coefficient. Let us consider a frame optimally designed at the ultimate limit state, i.e. a frame whose inter-story drift angle demand under the seismic intensity $C_{e,u}$ is equal to the assumed inter-story drift angle capacity (0.03 rad). By applying the ductility approximation ($\mu = q$), the drift angle demand under the design seismic forces must be equal to $0.03/q = 0.03/6 = 0.005$, with $q$ assumed equal to 6. The bold full line in Figure 5.1.3 is one possible curve representing the behaviour of the frame optimally designed at the ultimate limit state. 0.005 rad is exactly the inter-story drift angle capacity assumed for the serviceability limit state. Re-applying the ductility theory ($\mu = \nu$), the inter-story drift angle demand under moderate earthquakes can be predicted to be around $0.03/\nu = 0.03/2 = 0.015 >> 0.005$, being $\nu = 2$. Therefore, the serviceability limit state will be never satisfied for a steel MR frame optimally designed at the ultimate limit state. When the designer stiffens the structure in order to meet the prescribed EC8 serviceability requirement, the final design solution will be surely significantly over-resistant from a seismic point of view. It will reach the ultimate inter-story drift angle capacity under seismic intensities which are around three times the codified values, as shown in Figure 5.1.3 by the bold dash-pot line. On the contrary, under the codified seismic intensity relevant to the ultimate limit state of the structure ($C_{e,u}$) the inter-story drift angle demand will be around $0.03/3 = 0.05*2 = 0.01$ rad, as shown in Figure 5.1.3 where the bold dashed line is one possible structural solution meeting both the serviceability and the ultimate limit state requirements.
The Eurocode design rules are unique all over the world. In fact, there is no trace in other modern seismic design codes of situations similar to the European one. For example, according to the American UBC-94 (ICBO 1994), the serviceability requirement must be checked under the same forces used for the ultimate limit state design (those obtained by applying the strength reduction factor to the elastic design pseudo-acceleration spectrum). Moreover, the allowable value of the inter-story drift angle is practically the same as the European one. It must, indeed, be emphasised that it is very rare to have information about the non-structural damage control in modern seismic design standards. For example, in the more recent version of the UBC (ICBO 1997) the serviceability requirement has been removed, and it is clearly stated in the code that the specified design rules are developed only looking at the collapse prevention of structures. The same situation...
characterises other very recent structural design suggestions, such as those contained in the document FEMA 350 (2000). In the Building Standard Law of Japan the level 1 design approach prescribes the checking of allowable values of the inter-story drift angle similar to the European one. But the seismic forces used are 20% of those adopted for the ultimate limit state (Bruneau et al 1998), i.e. with a ratio between the serviceability and the ultimate limit state design forces significantly lower (1/5) than that implied by the EC8 suggestion (1/2).

Owing to both the contradictions of the EC8 design rules and the fact that it cannot be found any analogy within other design standards, the numerical study presented within this dissertation deals with two structural solutions. The first one has been obtained by neglecting the EC8 serviceability requirement and considering only the ultimate limit state checks. The second structural solution has instead been obtained by applying the EC8 serviceability requirement, using the value 0.006 rad as limiting inter-story drift angle. This value has been selected in order to contain the sizes of the frame members within the shapes commercially available in Europe. The first structural solution, will be indicated, in the following, by the symbol PMR-ULS (perimeter moment resisting – ultimate limit state), while the second solution will be symbolised by PMR-SLS (perimeter moment resisting – serviceability limit state). Figure 5.1.4 shows the frame member sizes of the frame PMR-ULS, while Figure 5.1.5 shows the dimensions of beams and columns of the frame PMR-SLS. It is apparent that the frame designed considering the serviceability limit state requirement is characterised by members sections significantly larger than those of the frame designed considering only the ultimate limit state requirements. It is worth noting that both two frames are designed considering the same gravitational loads, which are shown in Figure 5.1.6. Those gravitational loads are considered as contemporary acting with the ground acceleration time-histories successively applied to the structure. Gravitational loads have been obviously derived by applying the load combination factors suggested by EC8.
Figure 5.1.4. Member sizes of the frame PMR-ULS.

Figure 5.1.5. Member sizes of the frame PMR-SLS.
5.2 MODELLING OF FRAMES

5.2.1 General

The modelling assumptions, introduced in the numerical analysis of the frames described in Section 5.1, are discussed in this Section.

The elastic behaviour of both beams and columns has been modelled by means of the classical cubic interpolation of the transverse displacements of the element axis. Plasticity has been lumped at the ends of each beam-column element, using the well-known plastic hinge approximation. In particular, the effect of axial forces on bending strength is taken into account for columns using the simplified axial force (N) – bending moment (M) interaction diagram shown in Figure 5.2.1.1. However, plastic deformation has been assumed to develop only as a rotation in the plastic hinge, with no inelastic axial deformation. This is not theoretically correct, since it assumes a plastic flow along the bending moment axis only (see Figure 5.2.1.1), thus violating a well-known criterion of perfect plasticity according to which the plastic flow...
should be directed along the normal to the N-M interaction diagram. However, this assumption is frequently adopted by widely used numerical codes for seismic analysis of planar frames (DRAIN 2DX, Prakash & Powell 1993).

![Plastic flow vector (no axial inelastic deformation)](image)

**Figure 5.2.1.1. The simplified axial force (N) – bending moment (M) interaction diagram assumed in the analyses.**

Geometric non-linearity at the member level is considered only with reference to the global P-Δ effect, or chord-rotation effect. Then, the influence on the global response of local transverse displacements, starting from the rotated axis of each element (P-δ effect), has been neglected. See Chapter 3, for a deeper discussion about this approximation in the seismic analysis of frames. Strain-hardening for beam-columns is modelled by placing an elastic component in parallel with an inelastic component (Prakash & Powell 1993).

Beam-to-column connections have been modelled by means of inelastic springs, placed at the ends of each beam element. Strength and stiffness of connections have been selected in order to obtain rigid and full-strength connections. In Eurocode 3 (CEN 1993) the connection stiffness is characterised by means of a normalised rotational stiffness given by $k_\phi L/EI_b$, where $k_\phi$ is the connection rotational stiffness, $L$ the connected beam length, $E$ the Young modulus of the native material, $I_b$ the second moment of area of the transverse cross-section of the connected beam. According to Eurocode 3, beam-to-column steel connections can be considered rigid when the normalised rotational stiffness is greater than 25. In the numerical analyses carried out a value equal to 50 has been selected for this parameter. Full-strength connections have been assumed to have a plastic bending strength.
equal to the beam full plastic bending strength. In this case, to avoid numerical problems related to yielding of both the connection and the beam end at the same time, the beam plastic strength has been assigned a very high value. In this way, inelastic behaviour in the spring at the beam end is used to model plastic deformation in the beams rather than in connections. In fact, the use of such rigid and full-strength connections is devoted to investigate the behaviour of a continuous frame whose plastic hinges at the beam-ends are characterised by different hysteresis behaviours. The hysteresis behaviour has been changed from the elastic-perfectly plastic to the fully non-linear with negative stiffness branches in the moment-rotation relationship. This approach has allowed a simple modelling of the effects of local buckling phenomena in I-shaped beams subjected to strong plastic deformations. More detailed information about the hysteresis modelling of plastic zones in the frame is reported in the next sub-Section.

The column web panel zone of beam-to-column joints has been assumed to be always rigid and resistant enough to remain elastic. Moreover, no rigid beam end-offsets have been used, thus neglecting the reduction of beams and columns flexural stiffness due to the finite size of beam-to-column joints.

Figure 5.2.1.2 synthesises the modelling scheme of the beam-to-column joint area for the analysed frames.

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Figure 5.2.1.2. Schematic representation of frame modelling assumptions.
5.2.2 Modelling of the hysteresis behaviour of plastic hinges

As stated in previous Chapters, the seismic stability of inelastic structures is dominated by strength degradation phenomena in the yielding structural elements. In order to verify this also through numerical analyses, several strength-degrading mathematical models have been set-up for plastic hinges of the analysed frames. In particular, two types of strength degradation have been considered:

1. An "energy-controlled" strength degradation.
2. A "deformation controlled" strength degradation.

The term "energy controlled" strength degradation refers to the case where the bending strength of the generic plastic hinge has been reduced based on the total dissipated hysteretic energy. In particular, indicating by $M_0$ the initial (undamaged) bending strength of the plastic hinge and by $M_{0,\text{red}}$ its reduced (damaged) value, the following degradation rule has been applied:

$$M_{0,\text{red}} = M_0 \left(1 - \beta \frac{E_h}{M_0 \phi_{\text{ult}}}\right)$$

(5.2.2.1)

where $E_h$ indicates the hysteretic dissipated energy, $\phi_{\text{ult}}$ is an estimate of the hinge rotational capacity under monotonic loading conditions and, finally, $\beta$ is a damage rate controlling factor. The parameter $\beta$, to be calibrated based on experimental data, will be indicated in the following as 'Damage Factor' (see formula (5.2.2.1)). Really, $\phi_{\text{ult}}$ is an additional damage model parameter, which should be also defined on the basis of experimental evidence. Numerical values for both $\beta$ and $\phi_{\text{ult}}$, which have been used in the following numerical analyses, have been established on the basis of some experimental tests results available in the technical literature. Formula (5.2.2.1) has been applied to model strength degradation of bi-linear elasto-plastic rotational springs. This is shown in Figure 5.2.2.1, where the moment-rotation hysteretic rotational relationship depicted is the outcome of a numerical analysis and refers to a pre-selected plastic hinge of the analysed frame.

The term "deformation controlled" strength degradation refers instead to the modelling of strength degradation arising from an excessive maximum plastic rotation in the plastic hinge. In fact, one main cause of strength degradation in plastic hinges forming in I-shaped steel beams is the local buckling of the compressed parts of the cross section. Flange and web local
buckling, interacting both each other and with lateral-torsional buckling, can produce significant strength degradation in steel frames. The problem of modelling local buckling is quite complex and is not directly faced in this dissertation. In order to have a quantitative information about the role of local buckling-induced strength degradation, a simplified modelling approach has been undertaken. The monotonic moment-rotation relationship of the generic inelastic connection spring has been characterised by a maximum rotation activating a negative-slope moment-rotation branch, thus simulating local static instability (see Figure 5.2.2.2). The moment-rotation hysteretic relationship shown in Figure 5.2.2.2 is the outcome of a numerical analysis and refers to a pre-selected plastic hinge of the analysed frame.

**Figure 5.2.2.1.** Energy-controlled strength degradation for beam plastic hinges.

**Figure 5.2.2.2.** Deformation-controlled strength degradation for beam plastic hinges.
In order to explain better the modelling of local buckling-induced strength degradation, in Figure 5.2.2.3 a qualitative drawing is reported showing an hypothetical moment-rotation relationship. The bending moment developed during the stable part of the generic deformation excursion has been computed using the Richard and Abbot formula (1975). The rotation excursion leading to the activation of the softening branch ('Softening Rotation') has been set equal to the value of the rotation leading to the occurrence of this phenomenon during a monotonic loading condition (see Figure 5.2.2.3). Besides, as it can be seen in Figure 5.2.2.3, the reduced flexural strength in one direction has been set equal to the minimum value of the bending moment reached along the softening branch, during the whole preceding deformation history, in the same direction of loading.

\[ M \phi = \text{Softening Rotation} \]

\[ \phi_s = \text{Reduced bending strength} \]

*Figure 5.2.2.3. Deformation-controlled strength degradation for beam plastic hinges.*

The hysteresis models here briefly described have been implemented into a numerical code performing the step-by-step numerical integration of the equations of motion, based on the linear acceleration Newmark's method (1959). Viscous damping is modelled assuming that the viscous damping matrix is the linear combination of the mass and the stiffness matrix. The combination coefficients have been computed by assigning a 5% viscous damping ratio to the first and third elastic vibration periods of the structure.
5.3 ANALYSIS METHODOLOGY

There is a large agreement, within the seismic community, about the feasibility of using the inter-story drift angle ($\Delta/h$), i.e. the ratio between the displacement $\Delta$ of one floor relative to the adjacent lower one and the story height $h$, for measuring the frame seismic damage. In fact, the inter-story drift angle is, at the same time, easy to evaluate and physically meaningful. The inter-story drift angle could be related, through a simplified approach, to local plastic deformations (Gupta et al. 2000).

On the contrary, there is no agreement about the correct methodology for evaluating frames seismic performance. We have to recognise that this is a major and more general problem, which involves evaluation and analysis of every structural system. There are both seismological and structural aspects that can significantly affect the frame seismic performance.

From the seismological point of view, one main difficulty arises from a still partial understanding of the parameters that should be used for measuring the ground motion damage potential. Currently, the most dependable parameter we can use is the spectral elastic pseudo-acceleration ($S_{a,e}$, usually computed with reference to a 5% viscous damping ratio and for the first vibration mode of the structure) normalised by means of the gravity acceleration ($g$). Thus, we obtain the ratio between the maximum base shear that would act on the structure if it behaves linearly elastic (with a deformed shape according to the first mode of vibration) and the seismic weight. The first-mode spectral acceleration is the parameter usually adopted by seismic codes for establishing seismic hazard at a given site. This is the main reason why it has been decided to adopt spectral acceleration as ground motion shaking intensity measure. It is well known that the seismic response depends also, in a non-negligible way, on other ground motion parameters, such as the strong motion phase duration and the frequency content. However, when selecting the ground motion scaling factor, i.e. the ground motion damage potential parameter, it has to be considered that it is needed to know its value relevant to a given earthquake return period. In other words, it is necessary to have information about the probability of exceedance of a given level of that parameter at a given site. This information is available, at the time, only for spectral accelerations.

From the structural point of view, one major uncertainty is related to the type of hysteresis model used in the numerical analysis. In fact, looking at the
experimental results, it is apparent that the usually assumed elastic-perfectly-plastic hysteresis model is far from faithfully representing the cyclic response of plastic zones. Thus, it seems proper to wonder which is the effect of the type of hysteresis model adopted in the numerical analysis of the whole frame on the predicted response at different performance levels. This subject has been already deeply discussed in previous Chapters, where it has been shown, from a qualitative point of view, how strength degradation could affect the frame seismic response.

A synthetic picture of the frame seismic performance is frequently obtained by relating the ground motion damage potential (\(S_{a,e}/g\)) and the structure damage parameter. The latter is often identified with the maximum transient value of the inter-story drift angle (\(\Delta_{\text{max}}/h\)). The \(S_{a,e} \text{ vs } \Delta_{\text{max}}/h\) curve is usually obtained by scaling a given ground acceleration time-history at increasing values of the peak ground acceleration and computing the relevant maximum inter-story drift angle. By setting limiting values to the maximum inter-story drift angle, in relation with the selected limit state (serviceability, life safety, structural stability), it is possible to evaluate the earthquake intensity (\(S_{a,e}/g\)) inducing that damage level and to compare this intensity with that stipulated by the code, for that limit state and in relation with the site seismic hazard.

The methodology of analysis here briefly described has been recently proposed in the technical literature and is known with the name of “Incremental Dynamic Analysis” (IDA) (Hamburger et al. 2000, FEMA 2001). It could be also termed 'dynamic pushover', in order to contrast with the well-known 'static pushover' procedure. In the latter case, lateral story displacements are increased using a pre-fixed static pattern of increasing lateral forces, whilst in the former case lateral displacements are pushed over using a pre-fixed ground acceleration time-history scaled at increasing values of the peak ground acceleration. Figure 5.3.1 shows a qualitative picture of IDA curve. Some usual values of the limiting inter-storey drift angles, relevant to the Serviceability performance level (\(\Delta_{\text{Serviceability}}/h\)) and Life-Safety performance level (\(\Delta_{\text{Life-Safety}}/h\)) of MR steel frames, are also shown in the Figure. The limiting value relevant to the Structural Stability performance level is not reported in the Figure, since it is currently not considered by seismic codes. In Figure 5.3.1, three horizontal (dashed) lines are also reported, indicating three reference levels of spectral acceleration, which
should be stipulated by seismic codes in connection with the site seismic hazard. The level \( S_{a,e,\text{Ref}} \) should be achieved with inter-story drift angles lesser than or equal to the limiting value relevant to the Serviceability Limit State. From a dual perspective, it can be said that the non-structural damage limiting inter-story drift angle \( (\Delta_{\text{Serviceability}}/h) \) should be achieved under earthquakes having intensities (spectral accelerations) lesser than or equal to the reference one. Analogously, the reference level of spectral acceleration relevant to the Life-Safety performance level should be achieved with inter-storey drift angles lesser than or equal to the corresponding limit value \( (\Delta_{\text{Life-Safety}}/h) \), or, equivalently, the limiting value should be achieved under earthquakes having intensities lesser than the corresponding reference one. For example, in the qualitative case shown in Figure 5.3.1, the Serviceability and Life-Safety performance objectives have been achieved, whilst the Structural Stability performance level is attained under a too small spectral acceleration.

\[
S_{a,e} = S_{a,e,\text{SS-Ref}}, S_{a,e,\text{LS-Ref}}, S_{a,e,\text{S-Ref}}
\]

\[
(\Delta/h)_{\text{Serviceability}} = 0.005
\]

\[
(\Delta/h)_{\text{Life-Safety}} = 0.03
\]

\[
(\Delta/h)_{\text{Structural Stability}} = ?
\]

*Figure 5.3.1. A qualitative picture of IDA curve.*

It could be useful to remark that if the two parameters, which have been chosen as representative of seismic damage potential on one hand and of frame actual damage on the other hand, are the correct ones, then an increase
of the former should induce an increase of the latter. In other words, we should expect a monotonically increasing relationship between the ground motion damage potential parameter and the structural damage parameter. As it will be shown, this not always occurs. The following numerical analyses will show that, sometimes, an increase in the normalised spectral acceleration is associated with a constant or even decreasing maximum inter-story drift angle. We must conclude that, in those cases, the parameters chosen are not completely representative either of the earthquake damage potential or of the structure damage sensitivity, or of both of them. Nonetheless, the IDA methodology is a very useful tool, which allows us to highlight the influence on the predicted seismic response of both the hysteresis model adopted and the seismic input chosen. This is shown in the following, where several numerical analyses are presented and discussed.

A debatable assumption, often made when analysing structures subjected to earthquake ground motions, is the use of the peak ground acceleration (PGA) as earthquake intensity measure, instead of the first-mode spectral acceleration (S_{a,e}). The difference between a measure of safety in terms of PGA or in terms of S_{a,e} depends on the accelerogram used. This difference could be non-negligible, as it will be shown in the following numerical analyses. It is an open question if it is more appropriate to use the peak ground acceleration or the spectral acceleration. In fact, from one point of view the use of the spectral acceleration is more appropriate, as it allows the ratio of the structure first vibration period vs. earthquake dominant period to be taken into account. On the other hand, scaling a recorded ground acceleration time-history in such a way to match the codified elastic spectral acceleration implies often the use of accelerograms characterised by very large values of the PGA: are they realistic? In order to investigate quantitatively on the difference of a safety factor computed in terms of spectral acceleration or in terms of peak ground acceleration, both of them will be used as earthquake damage potential parameter in the following numerical analyses.

Another aspect, which should be further investigated, is that using the spectral acceleration (or also the peak ground acceleration) as earthquake intensity measure implies neglecting the effect of the earthquake duration on the seismic response of the structure. This aspect of the problem will not be faced in this dissertation. However, as will be explained better in the next
Section, the ground acceleration time-histories, which have been subsequently used in the numerical analyses, have been selected so that to be characterised by similar values of the strong motion phase duration.

It is contended that the problem of measuring the earthquake damage potential, as well as that of scaling it in order to measure safety at collapse of structures, is very important. In order to develop reliable performance-based seismic evaluation methodologies, it needs further developments.

5.4 SELECTED GROUND MOTIONS

Several numerical results concerning the seismic performance up until collapse (dynamic instability) of the frames described in Section 5.1, modelled according to Section 5.2 and analysed according to Section 5.3, will be shown in the following. The results have been obtained by the numerical integration of the equations of motion of the examined models of the frames, subjected to pre-selected ground acceleration time-histories. The ground motions have been taken from a recently developed European strong motion database (Ambraseys et al., 2000). It is well known that the type of ground acceleration time-history exerts a determinant influence on the seismic performance of structures. Peak ground acceleration, duration and frequency content are key factors influencing the response of a given structure. In this dissertation the 'dynamic pushover' approach, also called the IDA (incremental dynamic analysis) approach, is adopted for studying the frame seismic performance, as deeply discussed in the previous Section. Then, the peak ground acceleration ($PGA$) is assumed as the ground motion parameter to be scaled up until the occurrence of the structural collapse. The ground acceleration time histories have been chosen firstly fixing the earthquake. The well-known Campano-Lucano earthquake, sometimes called the Irpinia earthquake, which hit the South of Italy in 1980, has been chosen. Besides, the ground motions have been selected in order to have similar values of the 'effective' duration, as defined by Trifunac and Brady (1975) for measuring the strong motion phase duration. In fact, the four selected ground acceleration time-histories are characterised by similar values of the Trifunac duration, as can be seen in Table 5.4.1, where also peak ground accelerations can be read.
Table 5.4.1. Strong-motion phase duration of the selected ground motions.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Ground Acceleration Record</th>
<th>Name of the station</th>
<th>Direction of the component</th>
<th>PGA (g)</th>
<th>Trifunac Duration (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Campano-Lucano (1980)</td>
<td></td>
<td>Bagnoli-Irpino</td>
<td>EW</td>
<td>0.18</td>
<td>31.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bagnoli-Irpino</td>
<td>NS</td>
<td>0.14</td>
<td>41.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Storno</td>
<td>EW</td>
<td>0.32</td>
<td>38.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Storno</td>
<td>NS</td>
<td>0.22</td>
<td>40.02</td>
</tr>
</tbody>
</table>

Figures from 5.4.1, 5.4.2, 5.4.3 and 5.4.4 show the pseudo-acceleration elastic response spectrum of Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, StornoEW and StornoNS, respectively. A 5% viscous damping ratio has been assumed when computing these spectra. The spectral acceleration has been normalised by means of the gravity acceleration \((g = 9.81 \text{ m/s}^2)\). In the same Figures, the first-mode elastic vibration period \((T_1)\) and the 10/50 EC8 spectral acceleration \((S_{a,e,ULS-EC8})\) are also shown with reference to both the frame PMR-SLS and the frame PMR-ULS. Points of co-ordinates \((T_1, S_{a,e,ULS-EC8})\), which are indicated by circles in Figures from 5.4.1 to 5.4.4, are very important, because they represent the design point at the Ultimate Limit State. The ratio between the codified Ultimate Limit State spectral acceleration \((S_{a,e,ULS-EC8})\) and the computed spectral acceleration at the first-mode elastic vibration period of the structure, which will be referred to as the 'recorded' first-mode spectral acceleration \((S_{a,e,r})\), gives a measure of the scale factor that should be applied to the acceleration time-history in order to match the codified 10/50 spectral pseudo-acceleration. It is also useful to compare the ratio between the recorded peak ground acceleration (indicated as \(PGA_r\)) and its design value (indicated as \(PGA_{ULS-EC8}\)). Both the first-mode recorded spectral acceleration and the EC8 codified 10/50 value are summarised in Table 5.4.2, with reference to both the frame PMR-SLS and the frame PMR-ULS. Values of the recorded peak ground accelerations for the four ground acceleration time-histories selected have been already reported in Table 5.4.1. The ratios \(S_{a,e,r}/S_{a,e,ULS-EC8}\) and \(PGA_r/PGA_{ULS-EC8}\) are instead shown in Table 5.4.3.
Table 5.4.2. Recorded and 10/50 codified spectral accelerations.

<table>
<thead>
<tr>
<th></th>
<th>PMR-SLS</th>
<th>PMR-ULS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{a,e,r}/g$</td>
<td>0.28</td>
<td>0.24</td>
</tr>
<tr>
<td>$S_{a,e,ULS-EC8}$</td>
<td>0.633</td>
<td>0.14</td>
</tr>
<tr>
<td>Bagnoli-IrpinoEW</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>StornoEW</td>
<td>0.42</td>
<td>0.285</td>
</tr>
<tr>
<td>StornoNS</td>
<td>0.35</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 5.4.3. Ratios between recorded and codified ground motion parameters.

<table>
<thead>
<tr>
<th></th>
<th>$S_{a,e,r}/S_{a,e,ULS-EC8}$</th>
<th>$PGA_c/PGA_{ULS-EC8}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PMR-SLS</td>
<td>PMR-ULS</td>
</tr>
<tr>
<td>Bagnoli-IrpinoEW</td>
<td>0.45</td>
<td>0.83</td>
</tr>
<tr>
<td>Bagnoli-IrpinoNS</td>
<td>0.67</td>
<td>0.90</td>
</tr>
<tr>
<td>StornoEW</td>
<td>0.55</td>
<td>0.95</td>
</tr>
</tbody>
</table>

In order to explain the usefulness of Table 5.4.3, let us assume that the Seismic Stability Limit State is achieved under the $PGA_c/PGA_r$ scale factor of the ground acceleration time-history. For a sufficiently small value of viscous damping (5% is adequate), the following equality is well approximate:

$$\frac{PGA_c}{PGA_r} = \frac{S_{a,e,c}}{S_{a,e,r}}$$

(5.4.1)

Now, let us make the following hypothesis:

$$\frac{PGA_r}{PGA_{ULS-EC8}} > \frac{S_{a,e,r}}{S_{a,e,ULS-EC8}}$$

(5.4.2)

From inequality (5.4.2) the following alternative version can be easily derived:
\[
\frac{S_{a,e,\text{ULS-EC8}}}{PGA_{\text{ULS-EC8}}} > \frac{S_{a,e,r}}{PGA_r} \tag{5.4.3}
\]

which, remembering the equality (5.4.1), implies:

\[
\frac{S_{a,e,\text{ULS-EC8}}}{PGA_{\text{ULS-EC8}}} > \frac{S_{a,e,c}}{PGA_c} \quad \Rightarrow \quad \frac{PGA_c}{PGA_{\text{ULS-EC8}}} > \frac{S_{a,e,c}}{S_{a,e,\text{ULS-EC8}}} \tag{5.4.4}
\]

Then, we can write:

\[
\frac{PGA_r}{PGA_{\text{ULS-EC8}}} > \frac{S_{a,e,r}}{S_{a,e,\text{ULS-EC8}}} \quad \Rightarrow \quad \frac{PGA_c}{PGA_{\text{ULS-EC8}}} > \frac{S_{a,e,c}}{S_{a,e,\text{ULS-EC8}}} \tag{5.4.5}
\]

For example, looking in Table 5.4.3 at the row corresponding to Bagnoli-IrpinoEW and at the column relevant to the frame PMR-ULS, it can be seen that the ratio \(PGA_c/PGA_{\text{ULS-EC8}}\) is smaller than the ratio \(S_{a,e,c}/S_{a,e,\text{ULS-EC8}}\) (0.52 against 0.83). This implies that the ratio \(PGA_c/PGA_{\text{ULS-EC8}}\) will be smaller than the ratio \(S_{a,e,c}/S_{a,e,\text{ULS-EC8}}\). On the contrary, for the same accelerogram but considering the frame PMR-SLS, the ratio \(S_{a,e,c}/S_{a,e,\text{ULS-EC8}}\) will be smaller than the ratio \(PGA_c/PGA_{\text{ULS-EC8}}\), because we have a 0.45 value for the ratio \(S_{a,e,c}/S_{a,e,\text{ULS-EC8}}\), which is smaller than the ratio \(PGA_c/PGA_{\text{ULS-EC8}}\) equal to 0.52.
Figure 5.4.1. Pseudo-acceleration elastic response spectrum of Bagnoli-IrpinoEW (5% viscous damping ratio).

Figure 5.4.2. Pseudo-acceleration elastic response spectrum of Bagnoli-IrpinoNS (5% viscous damping ratio).
Figure 5.4.3. Pseudo-acceleration elastic response spectrum of SturnoEW (5% viscous damping ratio).

Figure 5.4.4. Pseudo-acceleration elastic response spectrum of SturnoNS (5% viscous damping ratio).
5.5 NUMERICAL RESULTS

5.5.1 Seismic stability based on the elastic – perfectly plastic hysteresis model

Numerical results concerning the seismic stability of both the frame designed considering the EC8 serviceability requirement (PMR-SLS) and the frame designed neglecting this requirement (PMR-ULS) are shown in this Section. Numerical results have been obtained using the elastic – perfectly plastic (EPP) model for representing the hysteresis behaviour of plastic hinges. Frame modelling assumptions, ground motions used and the analysis methodology have been deeply described in Sections 5.2, 5.3 and 5.4.

Figures 5.5.1.1 to 5.5.1.4 show the $S_{a,e}$ (first mode elastic spectral pseudo-acceleration, computed with reference to a 5% viscous damping ratio) vs. $\Delta_{\text{max}}/h$ (maximum transient inter-story drift angle) relationships, obtained using the four acceleration records described in Section 5.4. In the same figures, the EC8 codified maximum earthquake intensity relevant to the ultimate limit state of the structure ($S_{a,e,\text{ULS-EC8}}$), i.e. the EC8 elastic spectral pseudo-acceleration having a 10% probability of being exceeded in 50 years (return period equal to 475 years), is also shown. The $S_{a,e,\text{ULS-EC8}}$ earthquake intensity will be also indicated as the 10/50 spectral pseudo-acceleration in the following, with a symbolism taken from Gupta and Krawinkler (2000b). The $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ curves have been stopped when dynamic instability occurred.

To ascertain that divergence was not a numerical problem but a physical one, dynamic analyses were ran also for spectral acceleration values larger than the first one that caused divergence of the solution, verifying that for these larger values the structure was still unstable. Looking at Figures 5.5.1.1 to 5.5.1.4, a number of interesting observations can be made as summarised hereafter in the following paragraphs.

First of all, the criterion adopted for identifying dynamic instability will be discussed. It is a quite widespread opinion that the dynamic instability limit condition is as an asymptotic situation. Looking at Figure 5.5.1.4 one could be convinced that an upper bound to spectral acceleration exists in the form of an horizontal asymptote to the $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ curve. However, Figure 5.5.1.1 seems to indicate that dynamic instability occur when a finite critical value of
the drift angle demand is achieved, rather than an asymptotic situation. It is
deemed that the opinion that dynamic instability is reached as an asymptotic
condition has lead to a commonly adopted conventional way of defining the
stability limit value of spectral acceleration. In fact, dynamic instability is very
often identified by individuating the point of the $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ curve where
the slope of the tangent to the curve reduces to a pre-fixed small percentage of
the slope of the tangent at the origin of the same curve (Hamburger et al.
2000). The physical justification of this conventional method is that, for
spectral acceleration values larger than the one identified with the small
tangent slope criterion, a very small increase of the spectral acceleration itself
will induce a very large increase of the structure deformations. However, it is
contended that there is neither a theoretical nor a sufficiently verified
empirical basis to agree with the above opinion and to adopt the above
conventional criterion for identifying the stability limit condition. As shown in
Figure 5.5.1.1, there are cases where the slope of the curve is not so small at
the collapse point. Then, it could be possible to find situations where the slope
of the tangent to the $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ curve at the collapse point is larger than the
conventionally predefined small percentage of the slope of the tangent at the
origin of the same curve. Moreover, it can be observed that the transient inter-
story drift angle is not the only quantity that diverges at the collapse point. In
fact, let us to show to the reader the relationships between the elastic spectral
pseudo-acceleration and the maximum value of the residual inter-story drift
angle ($\Delta_r/h$), i.e. the maximum value of the permanent inter-story drift angle
computed among all the frame stories. Figures from 5.5.1.5 to 5.5.1.8 refer to
Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, StornoEW and StornoNS, respectively.
Also these curves have been stopped when dynamic instability occurred and it
can be clearly seen that not only the maximum transient but also the maximum
residual story displacements are faster increasing with the spectral
acceleration when the structure approaches collapse. In other words, when the
structure approaches dynamic instability, a significant reduction of the slope
of the tangent to the $S_{a,e}$ vs. $\Delta_r/h$ curve also occurs. Therefore, it seems proper
to wonder how the reduced tangent slope criterion should apply when
dynamic instability is judged looking at the residual inter-story drift angle
curves. In fact, the tangent at the origin of the $S_{a,e}$ vs. $\Delta_r/h$ curves has an
infinite slope and, then, any finite value is infinitesimal with respect to it!
Besides, in Chapter 2 it has been theoretically proved that dynamic instability of a simple SDoF system occurs when the drift demand reaches a finite critical value. In Chapter 2 it has been emphasised that the lateral strength of the relevant deformation mode is zero when the critical value of the drift angle is reached. In fact, in the limit situation, the destabilising fictitious lateral forces owing to the $P-\Delta$ effect are balanced by the internal lateral strength of the structure, and no lateral strength margin remains. In Chapter 3 it has also been shown that the concepts expressed with reference to SDoF systems can be extended to MDoF structures. However, it has already been emphasised in the same Chapter that it is significantly more difficult to identify a-priori the lateral deformation capacity of MDoF structures, since it is difficult to foresee the deformation mode at collapse of a given structure under a given ground acceleration time-history. Once the structure collapse mechanism is known, it would be very easy to compute the lateral deformation capacity of the structure and than to compare it with the deformation demand: when the latter quantity equals the former (i.e. when demand is equal to capacity) dynamic instability will occur. However, there are currently no methods able to predict the collapse mechanism of a given structure under a given earthquake, and then the above criterion cannot be applied, at the time, as a prediction tool, but only as a performance evaluation one. This aspect has been already emphasised in both Chapters 3 and 4, and will be not further discussed here.

In order to identify the dynamic stability limit state within the IDA approach, it is useful to notice that, when the structure is displaced laterally up until a zero-strength deformation and it is then left in that deformed configuration, the residual displacements will be exactly equal to the maximum one, i.e. no deformation recover will occur (see Section 2.2). Based on this observation, an approximate but simple criterion has been proposed in Chapter 2 to identify the stability limit state: dynamic instability will occur when the maximum residual inter-story drift angle is sufficiently close to the maximum transient inter-storey drift angle. The feasibility of this criterion is shown in Figures from 5.5.1.9 to 5.5.1.2, where the $S_{a,e}$ vs. $\Delta_{max}/h$ curves are superimposed to the $S_{a,e}$ vs. $\Delta/h$ ones, for each considered accelerogram. From the above figures it can be clearly seen that the maximum residual inter-story drift angle tends to increase and to become equal the maximum transient value. When the residual is sufficiently close to the maximum the structure is on the verge of
collapsing, i.e. it is at the dynamic stability limit state. Indeed, the equality between the residual and maximum drift can never be rigorously obtained, owing to residual viscous damping forces that damp out the dynamic response at the end of the earthquake, dissipating the residual kinetic energy. However, the difference is very small at the collapse point. A tolerance could be fixed in order to eliminate a certain degree of subjectivity implicit in this simplified methodology.

Figures from 5.5.1.13 to 5.5.1.16 show the same results of Figures from 5.5.1.9 to 5.5.1.12, but in terms of peak ground acceleration. The reader is referred to Section 5.3 for discussion about the use of the peak ground acceleration instead of the first mode spectral acceleration as earthquake damage potential measure.

In Figures from 5.5.1.17 to 5.5.1.20 the ratios \( S_{a,e}/S_{a,e,ULS-EC8} \) and \( PGA/PGA_{ULS-EC8} \) are plotted against the maximum transient inter-story drift angles up to the frame collapse point. As anticipated in Section 5.4, under the Bagnoli-IrpinoEW ground acceleration time-history the \( PGA/PGA_{ULS-EC8} \) ratio is higher than the \( S_{a,e}/S_{a,e,ULS-EC8} \) ratio, at any given structural performance level up until collapse, since the \( PGA/PGA_{ULS-EC8} \) ratio is larger than \( S_{a,e,r}/S_{a,e,ULS-EC8} \) ratio (see Table 5.4.3), \( PGA_r \) and \( S_{a,e,r} \) being the peak ground acceleration and the first mode spectral acceleration relevant to the recorded ground acceleration time-history. The same is true for the accelerograms SturnoEW and SturnoNS (see Figures 5.5.1.19, 5.5.1.20 and Table 5.4.3). On the contrary, in the case of Bagnoli-IrpinoNS, the \( PGA_r/PGA_{ULS-EC8} \) ratio is almost the same as the \( S_{a,e,r}/S_{a,e,ULS-EC8} \) ratio, thus implying that the safety measure in terms of peak ground acceleration is practically coincident with that in terms of spectral acceleration (see Figure 5.5.1.18 and Table 5.4.3). Figures 5.5.1.17 to 5.5.1.20 allow safety of the frame against dynamic instability at the 10/50 EC8 spectral acceleration level to be evaluated in a direct manner and from a quantitative point of view. Looking at these Figures, it is apparent that a quite large safety margin against the dynamic instability limit state of the frame named PMR-SLS exists, with respect to the codified 10/50 earthquake intensity level (spectral acceleration and/or peak ground acceleration). It is worth noting that the Structural Stability performance level should be related, for well-engineered new structures, to spectral acceleration levels larger than that used for the conventionally defined ultimate limit state,
i.e. larger than the 10/50 spectral pseudo-acceleration level. In fact, the $S_{a,e,ULS-EC8}$ spectral acceleration should correspond, as already noted in Section 5.3, to some structural performance level between 'damage control' (serviceability) and 'life safety'. Unfortunately, the author does not know the spectral acceleration levels corresponding to return periods larger than the 475 years value codified and written in Eurocode 8. At the author's knowledge there is no information available about this subject with reference to the Italian seismic territory. However, the ratios $S_{a,e}/S_{a,e,ULS-EC8}$ and $PGA/PGA_{ULS-EC8}$ are useful indicators of the frame safety against dynamic instability and the reader could establish by himself if the computed safety margins are to be considered as adequate or not. Obviously, the above ratios must be in any case larger than 1. Moreover, it is important to highlight that two aspects must be taken in mind when judging, in an absolute quantitative manner, these safety margins: plastic hinges in the frame have been modelled as rigid-plastic without any form of degradation and the frame is largely over-resistant owing to the serviceability requirement. The former aspect will be deeply discussed in the next Section, where results coming from the EPP non-degrading hysteresis model will be compared to those coming from the adoption of strength degrading hysteresis models for plastic hinges. The problem of the EC8 serviceability requirement implications on the frame performance at the ultimate limit state has already been discussed in Section 5.1. There it has been anticipated, through simple reasoning, that the satisfaction of the EC8 serviceability requirement, according to a standard design methodology, produces very stiff and strong steel frames. In order to help the reader to look at the frame PMR-SLS seismic performance up until the 10/50 EC8 spectral acceleration level, Figures from 5.5.1.21 to 5.5.1.24 show a zoom of Figures 5.5.1.9 to 5.5.1.12 into the relevant zone of the graphs. As can be seen, in all the cases examined, the inelastic drift demand under the EC8 10/50 spectral acceleration is significantly lower than the usually assumed allowable value compatible with negligible strength degradation in the structure (0.03 rad). In fact, the maximum transient inter-story drift angle demands under the 10/50 EC8 spectral acceleration are equal to 0.015, 0.011, 0.010 and 0.008 for Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, SturnoEW and SturnoNS, respectively. The average is 0.011 rad, which is significantly lesser than the limit value 0.03 rad. The average demand over capacity ratio, for the conventionally defined
ultimate limit state, is thus equal to 0.37 ($= 0.011/0.03$). The $S_{\text{a,e,SL}}$ spectral acceleration level, which should correspond to the serviceability requirement according to EC8 (see Section 5.3), is also shown in Figures from 5.5.1.21 to 5.5.1.24. As discussed in Sections 5.1 and 5.3, interpreting the EC8 written rules, the serviceability limit state spectral acceleration level could be interpreted as being equal to one half of the ultimate limit state spectral acceleration value: $S_{\text{a,e,SL}} = S_{\text{a,e,UL}}/2$. Accordingly, in Figures from 5.5.1.21 to 5.5.1.24 it can be seen that the maximum transient inter-story drift angle demands under the $S_{\text{a,e,SL}}$ spectral acceleration level are equal to 0.0050, 0.0050, 0.0058 and 0.0048 for Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, StornoEW and StornoNS, respectively. The average is 0.0052, which is quite close to the limit value of 0.006 rad. The average demand over capacity ratio, at the serviceability limit state, is thus equal to 0.87. Comparing the latter average value with the average demand over capacity ratio at the ultimate limit state (0.87 against 0.37), once again demonstrates that an EC8 designed steel moment resisting frame cannot satisfy, with the same level of confidence, both the serviceability and the ultimate limit state requirements. The ultimate limit state structure over-resistance obviously reflects in a higher safety factor against dynamic instability, too.

To better understand the influence of the serviceability requirement satisfaction on the seismic performance of EC8-designed structures, the frame PMR-ULS has been investigated. Results are synthesised with the help of Figures from 5.5.1.25 to 5.5.1.28, which are in terms of spectral acceleration, and of Figures from 5.5.1.29 to 5.5.1.32, which are instead in terms of peak ground acceleration. A close-up view of the seismic performance of the frame PMR-ULS, up until the 10/50 EC8 spectral acceleration level, is shown in Figures from 5.5.1.33 to 5.5.1.36. The inelastic maximum transient inter-story drift angle demands under the $S_{\text{a,e,UL}}$ earthquake intensity are equal to 0.022, 0.020, 0.033 and 0.025 for Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, StornoEW and StornoNS, respectively. The average of demand is 0.025 and, therefore, the average demand over capacity ratio is 0.83 ($= 0.025/0.03$). The inter-story drift angle demands at the serviceability limit state, i.e. under the $S_{\text{a,e,SL}}$ earthquake intensity, are instead equal to 0.012, 0.011, 0.010 and 0.013 for Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, StornoEW and StornoNS, respectively. The average of demand is equal to 0.0115 and, therefore, the
average demand over capacity ratio at the serviceability limit state is equal to 1.92. Thus, as well expected, the EC8 serviceability requirement is not satisfied but the structure is quite good designed at the ultimate limit state (compare the average demand over capacity ratio at the serviceability limit state with the same quantity referred to the ultimate limit state, i.e. 1.92 with 0.93). For a more direct comparison of the seismic performances of the two examined frames (PMR-SLS and PMR-ULS), the safety factors, both in terms of spectral acceleration \( \frac{S_{a,e}}{S_{a,e,ULS-EC8}} \) and in terms of peak ground acceleration \( \frac{PGA}{PGA_{ULS-EC8}} \), are shown for both frames in Figures from 5.5.1.37 to 5.5.1.40. Numerical values of the dynamic instability limit state safety factors are summarised in Table 5.5.1.1. With the exception of the spectral acceleration safety factor computed under the Bagnoli-IrpinoEW acceleration record, for each of the examined accelerograms, safety factors for the frame PMR-ULS were significantly lower than safety factors for the frame PMR-SLS. The exception of Bagnoli-IrpinoEW acceleration record is justified looking at its elastic response pseudo-acceleration spectrum, represented in Figure 5.4.1. It can be observed that an increase of the vibration period of the frame PMR-SLS, caused by yielding during seismic response, produces an increase of pseudo-acceleration, whilst an increase of the vibration period is favourable for the frame PMR-ULS, since it produces a decrease of the spectral ordinate. When changing the structure, there is a change in both the numerator and the denominator of the ratio \( \frac{S_{a,e}}{S_{a,e,ULS-EC8}} \). The numerator is smaller for the frame PMR-ULS than for the frame PMR-SLS, thus representing the fact that the frame PMR-ULS is weaker than the frame PMR-SLS. But also the \( S_{a,e,ULS-EC8} \) limiting value is smaller for the frame PMR-ULS than for the PMR-SLS one. Then, it can be argued that the decrease in the absolute value of the spectral acceleration leading to collapse the frame PMR-ULS with respect to the one leading to collapse the frame PMR-SLS has not been sufficient, in the case of Bagnoli-IrpinoEW, to compensate the decrease of the codified 10/50 spectral acceleration level. This could just be justified considering that particular shape of the Bagnoli-IrpinoEW pseudo-acceleration elastic response spectrum. On the contrary, for all the others accelerograms, there are no significant differences in the shape of the pseudo-acceleration response spectrum in the right neighbourhood of the first mode vibration period of both the frame PMR-SLS and the frame PMR-ULS.
Obviously, the design value of the peak ground acceleration at the ultimate limit state ($PGA_{ULS-EC8}$) is not affected by the structure properties. Consequently, the safety factors in terms of $PGA$ for the frame PMR-SLS are always larger than the ones related to the frame PMR-ULS. In Table 5.5.1.1, it can be seen that the mean values of the safety factors of the frame PMR-ULS are significantly lower than the safety factors of the frame PMR-SLS, both in terms of spectral acceleration (2.75 against 4.15) and in terms of peak ground acceleration (2.18 against 4.85).

Table 5.5.1.1. Safety factors against dynamic instability for the two frames examined.

<table>
<thead>
<tr>
<th></th>
<th>$S_{a,e,c}/S_{a,e,ULS-EC8}$</th>
<th>$PGA_c/PGA_{ULS-EC8}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PMR-SLS</td>
<td>PMR-ULS</td>
</tr>
<tr>
<td>Bagnoli-IrpinoEW</td>
<td>3.7</td>
<td>4.3</td>
</tr>
<tr>
<td>Bagnoli-IrpinoNS</td>
<td>5.0</td>
<td>1.8</td>
</tr>
<tr>
<td>StornoEW</td>
<td>4.0</td>
<td>1.7</td>
</tr>
<tr>
<td>StornoNS</td>
<td>3.9</td>
<td>3.2</td>
</tr>
<tr>
<td>Average</td>
<td>4.15</td>
<td>2.75</td>
</tr>
</tbody>
</table>

In Figures from 5.5.1.41 to 5.5.1.48, the application of the lateral force-based ‘Damage Index’ presented in Chapter 4 (see formula 4.2.8) is shown with reference to both the frame PMR-SLS and the frame PMR-ULS. The index has been computed according to its definition presented in Chapter 4. Then, the following procedure has been applied:

2. Extraction from the output of the residual lateral story displacements.
3. Computation of the residual inter-storey drift angles.
4. Transformation of the residual inter-storey drift angles into residual fictitious lateral forces, through the use of the geometric stiffness matrix. The sum of these forces gives us the 'residual base shear'. Normalising each residual lateral story force by means of the residual base shear, yields the normalised residual lateral force pattern.
5. Execution of a first-order incremental static analysis under the residual lateral force pattern computed at the previous step. The base shear relevant to the formation of a plastic collapse mechanism is the post-
earthquake lateral strength to be considered for measuring closeness of the structure to dynamic instability.

6. Computation of the ratio between the 'residual base shear' (computed at step 4) and its first-order plastic collapse value (computed at step 5). This ratio is the 'Damage Index' defined at Chapter 4.

It is worth remembering that we are dealing with the behaviour of a frame whose plastic hinges have been characterised by the elastic-perfectly plastic hysteretic behaviour. Then, there is no strength degradation. Moreover, it is important to remember that, as explained in Chapter 4, the approach based on the computation of the first-order plastic strength is allowed if and only if the structure has already formed a plastic motion mechanism before the achievement of the global stability limit state. This condition has been always satisfied in our cases and it is deemed to be usual in case of building frame structures, owing to the low level of vertical loading.

As can be seen looking at Figures from 5.5.1.41 to 5.5.1.48, the proposed Damage Index ranges from 0 to 1 (collapse). The numerical accuracy of the IDA analysis methodology in capturing the Seismic Stability Limit State, i.e. in yielding a Damage Index equal to 1, is related to the step used for scaling the considered ground motion. In the analyses carried out within this dissertation a step size equal to 0.2 times the recorded peak ground acceleration was adopted, unless otherwise specified. This explains why the Damage Index did not reach a value exactly equal to 1. Diminishing the step size would produce a Damage Index closer to 1 when the structure approaches collapse. However, from the point of view of the evaluation of safety of the frame against dynamic instability, the level of spectral acceleration leading to collapse is to be considered the first objective of the numerical analysis. In the examined cases, a very small increase of the largest values computed for spectral accelerations would be required to yield a Damage Index equal to 1. Thus, considering also that the numerical analyses presented in this Section were based on the simplified assumption of an elastic-perfectly-plastic hysteresis behaviour for plastic hinges, the above increase was considered to be negligible. In the next Section, the paramount role of strength degradation in determining the level of spectral acceleration producing the achievement of the Seismic Stability Limit State will be shown.
Figure 5.5.1.1. $S_{a,e}$ vs. $\Delta_{max}/h$: frame PMR-SLS; model EPP; record Bagnoli-IrpinoEW.

Figure 5.5.1.2. $S_{a,e}$ vs. $\Delta_{max}/h$: frame PMR-SLS; model EPP; record Bagnoli-IrpinoNS.
Figure 5.5.1.3. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$: frame PMR-SLS; model EPP; record SturnoEW.

Figure 5.5.1.4. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$: frame PMR-SLS; model EPP; record SturnoNS.
Figure 5.5.1.5. $S_{a,e}$ vs. $\Delta_e/h$: frame PMR-SLS; model EPP; record Bagnoli-IrpinoEW.

Figure 5.5.1.6. $S_{a,e}$ vs. $\Delta_e/h$: frame PMR-SLS; model EPP; record Bagnoli-IrpinoNS.
Figure 5.5.1.7. $S_{a,e}$ vs. $\Delta/h$: frame PMR-SLS; model EPP; record SturnoEW.

Figure 5.5.1.8. $S_{a,e}$ vs. $\Delta/h$: frame PMR-SLS; model EPP; record SturnoNS.
Figure 5.5.1.9. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_c/h$: frame PMR-SLS; model EPP; record Bagnoli-IrpinoEW.

Figure 5.5.1.10. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_c/h$: frame PMR-SLS; model EPP; record Bagnoli-IrpinoNS.
Figure 5.5.11. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_{e}/h$: frame PMR-SLS; model EPP; record SturnoEW.

Figure 5.5.12. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_{e}/h$: frame PMR-SLS; model EPP; record SturnoNS.
Figure 5.5.1.13. PGA vs. $\Delta_{\text{max}}/h$ and $\Delta_/h$: frame PMR-SLS; model EPP; record Bagnoli-IrpinoEW.

Figure 5.5.1.14. PGA vs. $\Delta_{\text{max}}/h$ and $\Delta_/h$: frame PMR-SLS; model EPP; record Bagnoli-IrpinoNS.
Figure 5.5.1.15. PGA vs. $\Delta_{\text{max}}/h$ and $\Delta_r/h$: frame PMR-SLS; model EPP; record SturnoEW.

Figure 5.5.1.16. PGA vs. $\Delta_{\text{max}}/h$ and $\Delta_r/h$: frame PMR-SLS; model EPP; record SturnoNS.
Figure 5.5.1.17. Comparison of safety factors ($S_{a,e}/S_{a,e,ULS-EC8}$ and $PGA/PGA_{ULS-EC8}$): frame PMR-SLS; model EPP; record Bagnoli-IrpinoEW.

Figure 5.5.1.18. Comparison of safety factors ($S_{a,e}/S_{a,e,ULS-EC8}$ and $PGA/PGA_{ULS-EC8}$): frame PMR-SLS; model EPP; record Bagnoli-IrpinoNS.
Figure 5.5.1.19. Comparison of safety factors ($S_{a,e}/S_{a,e,ULS-EC8}$ and $PGA/PGA_{ULS-EC8}$): frame PMR-SLS; model EPP; record SturnoEW.

Figure 5.5.1.20. Comparison of safety factors ($S_{a,e}/S_{a,e,ULS-EC8}$ and $PGA/PGA_{ULS-EC8}$): frame PMR-SLS; model EPP; record SturnoNS.
Figure 5.5.1.21. A close-up view of the seismic performance of the frame PMR-SLS up until the EC8 codified 10/50 spectral acceleration: model EPP, record Bagnoli-IrpinoEW.

Figure 5.5.1.22. A close-up view of the seismic performance of the frame PMR-SLS up until the EC8 codified 10/50 spectral acceleration: model EPP, record Bagnoli-IrpinoNS.
Figure 5.5.1.23. A close-up view of the seismic performance of the frame PMR-SLS up until the EC8 codified 10/50 spectral acceleration: model EPP, record SturnoEW.

Figure 5.5.1.24. A close-up view of the seismic performance of the frame PMR-SLS up until the EC8 codified 10/50 spectral acceleration: model EPP, record SturnoNS.
Here dynamic instability occurred

Figure 5.5.1.25. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_c/h$: frame PMR-ULS; model EPP; record Bagnoli-IrpinoEW.

Here dynamic instability occurred

Figure 5.5.1.26. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_c/h$: frame PMR-ULS; model EPP; record Bagnoli-IrpinoNS.
Figure 5.5.1.27. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_{e}/h$: frame PMR-ULS; model EPP; record SturnoEW.

Figure 5.5.1.28. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_{e}/h$: frame PMR-ULS; model EPP; record SturnoNS.
Here dynamic instability occurred

Figure 5.5.1.29. PGA vs. $\Delta_{\text{max}}/h$ and $\Delta_{\text{r}}/h$: frame PMR-ULS; model EPP; Bagnoli-IrpinoEW.

Here dynamic instability occurred

Figure 5.5.1.30. PGA vs. $\Delta_{\text{max}}/h$ and $\Delta_{\text{r}}/h$: frame PMR-ULS; model EPP; Bagnoli-IrpinoNS.
Figure 5.5.1.31. PGA vs. $\Delta_{\text{max}}/h$ and $\Delta_r/h$: frame PMR-ULS; model EPP; SturnoEW.

Figure 5.5.1.32. PGA vs. $\Delta_{\text{max}}/h$ and $\Delta_r/h$: frame PMR-ULS; model EPP; SturnoNS.
Figure 5.5.1.33. A close-up view of the seismic performance of the frame PMR-ULS up until the EC8 codified 10/50 spectral acceleration: model EPP, record Bagnoli-IrpinoEW.

Figure 5.5.1.34. A close-up view of the seismic performance of the frame PMR-ULS up until the EC8 codified 10/50 spectral acceleration: model EPP, record Bagnoli-IrpinoNS.
Figure 5.5.1.35. A close-up view of the seismic performance of the frame PMR-ULS up until the EC8 codified 10/50 spectral acceleration: model EPP, record StornoEW.

Figure 5.5.1.36. A close-up view of the seismic performance of the frame PMR-ULS up until the EC8 codified 10/50 spectral acceleration: model EPP, record StornoNS.
Figure 5.5.1.37. Comparison of safety factors of the frame PMR-ULS with safety factors of the frame PMR-SLS (model EPP; record Bagnoli-IrpinoEW).

Figure 5.5.1.38. Comparison of safety factors of the frame PMR-ULS with safety factors of the frame PMR-SLS (model EPP; record Bagnoli-IrpinoNS).
Figure 5.5.1.39. Comparison of safety factors of the frame PMR-ULS with safety factors of the frame PMR-SLS (model EPP; record SturnoEW).

Figure 5.5.1.40. Comparison of safety factors of the frame PMR-ULS with safety factors of the frame PMR-SLS (model EPP; record SturnoNS).
Figure 5.5.1.41. $S_{a,e}$ vs. Damage Index: frame PMR-SLS; model EPP; record Bagnoli-IrpinoEW.

Figure 5.5.1.42. $S_{a,e}$ vs. Damage Index: frame PMR-SLS; model EPP; record Bagnoli-IrpinoNS.
Figure 5.5.1.43. $S_{a,e}$ vs. Damage Index: frame PMR-SLS; model EPP; record SturnoEW.

Figure 5.5.1.44. $S_{a,e}$ vs. Damage Index: frame PMR-SLS; model EPP; record SturnoNS.
Figure 5.5.1.45. $S_{a,e} vs. \text{Damage Index: frame PMR-ULS; model EPP; record Bagnoli-IrpinoEW.}$

Figure 5.5.1.46. $S_{a,e} vs. \text{Damage Index: frame PMR-ULS; model EPP; record Bagnoli-IrpinoNS.}$
Figure 5.5.1.47. $S_{a,e}$ vs. Damage Index: frame PMR-ULS; model EPP; record SturnoEW.

Figure 5.5.1.48. $S_{a,e}$ vs. Damage Index: frame PMR-ULS; model EPP; record SturnoNS.
5.5.2 **Seismic stability based on strength-degrading hysteresis models**

Numerical results coming from the use of strength-degrading hysteresis models are compared in this Section with those shown in the previous one, which were derived using the elastic – perfectly plastic hysteresis behaviour for plastic hinges (no degradation of mechanical properties). A general overview of the hysteresis model developed and implemented has already been presented in Section 5.2.2. As already emphasised in previous discussions (see Sections 2.3 and 3.3), it is expected that the Seismic Stability Limit State is significantly affected by strength degradation and the main aim of this section is to verify this assertion through numerical analyses.

It is worth emphasising that, in general, both strength and stiffness degradations are present in a structural component subjected to an inelastic deformation history. According to the experimental evidence, stiffness degradation has various forms. With reference, for example, to the moment–rotation relationship of a beam plastic zone, there could be a reduction of the slope of the unloading branch (elastic stiffness) or also a reduction of the tangent stiffness during a loading phase owing to a local static instability phenomenon. Both two types of stiffness degradation are important phenomena, but the latter (stiffness degradation caused by local static instability) is more directly influencing strength degradation and, then, the seismic stability of the whole structure. In fact, degradation of the unloading stiffness influences the achievement of the Seismic Stability Limit State only in an indirect manner, since it does not affect deformation capacities but modifies deformation demands. On the contrary, stiffness degradation owing to a local instability phenomenon directly influences both demand and capacity, since this type of stiffness degradation is naturally accompanied by significant strength degradation. Strength degradation is present in various forms, too. In fact, there could be strength degradation related to the achievement of excessive plastic deformation in one direction or also strength degradation due to repetition of plastic deformations of given amplitude. The influence of both two types of strength degradation on the seismic stability of the selected steel frames is investigated in the following. In particular, in order to both simplify the hysteresis models and understand the role of each type of strength degradation, it has been decided to adopt two ideal types of hysteresis behaviours. Firstly, a model characterised only by energy-controlled strength
degradation has been assumed and, secondly, a model characterised only by deformation-controlled strength degradation has been adopted. As better explained in Section 5.2.2, the term 'energy-controlled' strength degradation refers to the case where the reduction in the local strength of a plastic hinge is taken as a function of the hysteretic dissipated energy. On the contrary, the term 'deformation-controlled' strength degradation refers to the case where the reduction in strength is dictated by the maximum plastic deformation developed in the plastic hinge throughout its deformation history. More precise information about the modelling assumptions, needed to represent both two forms of strength degradation, has been given in Section 5.2.2. Deformation-controlled strength degradation is very frequent for steel beam plastic hinges, being the result of local buckling phenomena. In comparison, it is deemed that the effect of energy-related strength degradation (meaning degradation induced by repetition of plastic deformations of a given amplitude) is relatively less important. However, the energy-related strength-degradation could be also taken as an ideal model, through which simulate adequate strength degradation and obtain correct evaluation of stability conditions of the whole structure.

It is contended that an accurate and reliable evaluation of the seismic stability of structures can be obtained only using hysteresis models able to take account of both stiffness and strength degradation. In the following paragraphs, the effects of energy-controlled and deformation-controlled strength degradation will be separately discussed.

As made in the previous Section with reference to the elastic – perfectly plastic hysteresis model, results concerning the frame designed considering the EC8 serviceability requirement (PMR-SLS) will be firstly shown and then compared to those concerning the frame designed neglecting the above requirement (PMR-ULS).

Figures from 5.5.2.1 to 5.5.2.4 illustrate the results obtained by the analysis of the seismic response of the frame PMR-SLS subjected to the four selected accelerograms (Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, SturnoEW and SturnoNS, respectively). In these Figures, the $S_{a,e}$ (first mode elastic spectral pseudo-acceleration, computed with reference to a 5% viscous damping ratio) vs. $\Delta_{\text{max}}/h$ (maximum transient inter-story drift angle) curves have been superimposed to the $S_{a,e}$ vs. $\Delta_r/h$ (maximum residual inter-story drift angle)
relationships. Two levels of energy-controlled strength degradation have been assumed. These two levels are characterised by means of the numerical value given to a special model parameter, here named 'Damage Factor'. A zero value of the 'Damage Factor' implies no strength degradation, i.e. the assumption of an elastic – perfectly plastic (EPP) hysteresis behaviour of plastic hinges. The EPP model is the reference one, since it is largely used in the inelastic analysis of steel frames. A 'Damage Factor' equal to 0.15 has been also used in the numerical study, thus representing strength degradation related to the repetition of plastic deformations. The latter value has been selected, together with other values of the hysteresis model parameters, based on the results coming from some experimental results on steel members and joints that were available. It is deemed that the value 0.15 is to be considered as an upper bound in the case of steel beam-to-column joints. It is important to emphasise that this value is simply a target and it has not been selected to precisely represent strength degradation in our frame plastic hinges. The main aim of the use of a non-zero value for the 'Damage Factor' is to study the effect of energy-related strength degradation on the frame seismic response, at different performance levels, in order to capture quantitatively its potential influence.

Figures from 5.5.2.1 to 5.5.2.4 show that, in the case of the frame PMR-SLS, strength degradation is definitely not influent for high performance levels (from serviceability to life-safety), whilst its importance could be formidable for reliably evaluating the earthquake intensity leading the structure to collapse. In fact, both the maximum transient and maximum residual inter-story drift angle is initially negligibly affected by the value of the 'Damage Factor', but both of them strongly increase with the level of strength degradation (increasing value of the 'Damage Factor') when the structure approaches collapse. The elastic spectral pseudo-acceleration over which the effect of strength degradation becomes significant has been resulted higher than the EC8 10/50 spectral acceleration, for all the four accelerograms considered. Therefore, at least in the case of the frame PMR-SLS, the adoption of an elastic – perfectly plastic hysteresis model appears to be definitely adequate for the analysis of the frame seismic performance up until the codified 10/50 spectral acceleration level. On the contrary, the use of more refined hysteresis models appears to be a necessary pre-requisite of the frame modelling, in order to obtain realistic predictions of the earthquake intensity...
leading the structure to collapse during the earthquake. When judging these numerical results, it should be reminded that the frame PMR-SLS is strongly over-resistant due to the serviceability requirement satisfaction, as already noted. Therefore, it should be considered that the plastic hinges’ strength degradation is initially compensated by the design over-strength, thus leading to high critical values of spectral accelerations over which strength degradation itself becomes significant. This is one main reason for analysing also the frame PMR-ULS, which is characterised by a minimum design over-strength.

Figures from 5.5.2.5 to 5.5.2.8 show both the $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $S_{a,e}$ vs. $\Delta_r/h$ curves obtained when analysing the frame PMR-ULS, under the Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, StunoEW and StunoNS acceleration records, respectively. Among these Figures, the case of StunoNS and a Damage Factor equal to 0.15 (Figure 5.5.2.8) should be soon discussed, since it appears to be different from all the others numerical results. In this particular case, the empirical stability criterion $\Delta_{\text{max}} = \Delta_r$, which has been previously proposed, seems to be hardly applicable. In fact, the numerical solution diverges before the maximum residual and maximum transient values become sufficiently close each other. Is this simply a numerical problem? An answer to this question will be attempted later, after showing also numerical results deriving from the adoption of deformation-controlled strength degradation. The reader is invited to keep in mind the problem, skipping it when looking at Figure 5.5.2.8. As it can be seen, for the frame PMR-ULS too, the assumed level of strength degradation in beams’ plastic hinges produced a negligible effect on deformation demands for elastic spectral accelerations up until the codified 10/50 value ($S_{a,e,\text{ULS-EC8}}$). It is apparent that higher strength degradation in the frame (i.e. higher values of the ‘Damage Factor’) will induce larger deformation demands at each level of spectral acceleration. Then, with higher levels of strength degradation in the frame, the earthquake intensity level over which strength degradation itself becomes an important phenomenon could become smaller than the 10/50 spectral acceleration level. However, as already said, the 0.15 value assumed for the Damage Factor is deemed to be an upper bound for representing energy-controlled strength degradation of steel beam plastic hinges. Moreover, it should be considered that the frame PMR-ULS was designed neglecting the serviceability requirement, thus
obtaining a sample steel frame with the minimum required seismic strength. Then, it is concluded that the energy-related strength degradation can be considered as a negligible phenomenon in steel frames subjected to earthquakes having intensities up until the 10/50 spectral acceleration level, which is used in the design phase for checking the conventional ultimate limit state. It is important to highlight that this conclusion is well expected, since a well-engineered steel frame should experience, under the 10/50 spectral acceleration level, deformation demands less than or equal to 0.03 rad, if they are synthetically measured in terms of inter-story drift angle. This limiting value is a target representing a bound beyond which strength degradation becomes locally significant. Therefore, numerical results obtained here confirm also the validity of the ultimate-limit-state design criterion usually adopted for steel frames.

The application of the Damage Index formulation proposed in Chapter 4 with reference to strength-degrading hysteresis behaviours is shown in Figures from 5.5.2.9 to 5.5.2.16. In particular, Figures from 5.5.2.9 to 5.5.2.12 refer to the frame PMR-SLS subjected to the Bagnoli-IrpinoEW, Bagnoli-IrpinoNS, StornoEW and StornoNS acceleration records, respectively. Figures from 5.5.2.9 to 5.5.2.12 are analogous, but they refer to the frame PMR-ULS. The procedure detailed in the previous Section has been applied, using the residual flexural strength of the plastic hinges at the end of the earthquake for computing the plastic collapse value of the residual base shear force. As already emphasised in previous discussions, this approach is valid if and only if a plastic motion mechanism has been activated prior to the achievement of the dynamic stability limit condition. This was the case for the examined frames and ground motions. Looking at the $S_{a,e}$ vs. Damage Index curves, it can be observed the very strong impact of local strength degradation in determining the achievement of the Seismic Stability Limit State. In fact, for every spectral acceleration level, the Damage Index computed using a strength-degrading hysteresis model (Damage Factor = 0.15) is significantly larger than the same quantity computed using a non-degrading model (Damage Factor = 0). From a dual perspective, it can also be said that the spectral acceleration inducing a given level of structural damage (i.e. a given value of the Damage Index) is significantly lesser for the strength-degrading case than for the elastic-perfectly plastic hysteresis model. Once again, it can
be concluded that safety at collapse of MR steel frames cannot be reliably evaluated basing the analysis on the elastic-perfectly plastic hysteresis model. The development of reliable mathematical models of the actual hysteresis behaviour of plastic zones is the necessary prerequisite for a reliable computation of safety of steel frames against the Seismic Stability Limit State. It is emphasised that this conclusion is not banal, because it is derived for a specific class of structures, where the level of vertical loading is relatively small with respect to the building lateral strength. As shown in Chapter 2, for systems characterised by high levels of vertical loading and/or high levels of local ductility the global instability of the structure could occur before the achievement of significant local degradation.

Figures from 5.5.2.17 to 5.5.2.24 show the effect of local softening (deformation-controlled strength degradation) on the computed frame performance. Beams' plastic hinges in the frame have been equipped with a restoring moment-rotation relationship characterised by the occurrence of an unstable (negative stiffness) branch. The negative stiffness branch in the moment-rotation relationship of the generic plastic hinge takes place when a limiting value of the rotation excursion is achieved. The limiting rotation excursion is indicated as 'Softening Rotation' (see Figure 5.2.2.3). Two values of this model parameter have been used in the numerical analyses, namely 0.03 rad, representing a target level of the 'Softening Rotation', and 0.01 rad, which simulates a premature appearance of local static instability.

In Figures from 5.5.2.17 to 5.5.2.20 results concerning the frame PMR-SLS are shown. First of all, it can be observed again the negligible influence of strength degradation on deformation demands up until the achievement of spectral accelerations larger than the codified EC8 10/50 level. Then, the main conclusion derived in the previous paragraphs about the suitability of the elastic-perfectly plastic hysteresis model for high level structural performances (let say up to the 'Life-Safety' level) seems to be confirmed. Secondly, we can see again a relatively large effect on the 'collapse spectral acceleration', meaning the maximum spectral acceleration that the frame was able to sustain before dynamic instability occurred. The effect of deformation-controlled strength degradation appears to be less important with respect to the effect of energy-controlled strength degradation, which has been previously shown and discussed (see Figures from 5.5.2.1 to 5.5.2.4). But, it is to be
highlighted that the quantitative role of strength degradation strongly depends by the type of collapse mechanism or, more generally speaking, by the type of energy dissipation path through the frame. In fact, as specified in Section 5.2, only plastic hinges in beams have been characterised by strength-degrading hysteresis models, whilst plastic hinges in columns have been modelled using the elastic-perfectly plastic hysteresis behaviour. Then, it is to be expected that, in the numerical analyses carried out, the effect of strength degradation is quantitatively less important when the collapse mechanism is a storey one. This is the case of the results shown in Figures from 5.5.2.17 to 5.5.2.20, where six plastic hinges formed at the base of the frame and at the top of the columns on the second floor, thus producing a story plastic motion mechanism.

In Figures from 5.5.2.21 to 5.5.2.24 numerical results concerning the seismic response of the frame PMR-ULS are shown. Once again, plastic hinges in beams have been characterised by deformation-controlled strength degradation, using the same 'Softening Rotation' values previously adopted for studying the frame PMR-SLS. As it can be seen, when a 'Softening Rotation' equal to 0.01 rad has been selected, the numerical solution diverged previous that the empirical criterion of the residual displacement approaching the maximum one was adequately satisfied. This problem occurred already in the case of Figure 5.5.2.8, as previously noted. In order to understand the reason of this type of sharp divergence of the lateral displacements, it is useful to consider the limit case of a brittle structure, which is shown in the following Figure (not numbered for convenience).

![Figure. The special case of a brittle structure behaviour.](image-url)
It can be argued that, in the limit case of a brittle structure, the transition from vibration to drift in one direction occurs sharply. In fact, up until the point of brittle rupture the residual lateral displacements are zero, since the structure has an elastic behaviour. When the maximum internal strength is reached the system immediately reaches its stability limit state, without any prior plastic deformation. In the case of a Softening Rotation equal to 0.01 rad, the local plastic deformation capacity is relatively small and the frame response could be quite close to the limit situation of a brittle structure. For the frame PMR-SLS, where a storey plastic motion mechanism developed, the sensitivity to local instability in beams' plastic hinges was smaller than it was in the case of the frame PMR-ULS. This could explain why the problem of the sharp divergence of lateral displacements occurred only for the PMR-ULS frame. It is remarked that the main concern of the current dissertation is the study of ductile systems, where strength degradation takes place only after a significant plastic deformation has been developed before instability occurs.

As it can be seen in Figures from 5.5.2.21 to 5.5.2.24, the premature appearing of local static instability could be a very dangerous phenomenon for structures without significant over-strength. This phenomenon could lead the structure to abruptly collapse under earthquakes having intensities (spectral accelerations) comparable or even lesser then the codified 10/50 spectral accelerations.
**Figure 5.5.2.1.** $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_{e}/h$: influence of energy-controlled strength degradation (frame PMR-SLS; record Bagnoli-IrpinoEW).

**Figure 5.5.2.2.** $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_{e}/h$: influence of energy-controlled strength degradation (frame PMR-SLS; record Bagnoli-IrpinoNS).
Figure 5.5.2.3. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_r/h$: influence of energy-controlled strength degradation (frame PMR-SLS; record SturnoEW).

Figure 5.5.2.4. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_r/h$: influence of energy-controlled strength degradation (frame PMR-SLS; record SturnoNS).
Figure 5.5.2.5. $S_{a,e}$ vs. $\Delta_{max}/h$ and $\Delta_e/h$: influence of energy-controlled strength degradation (frame PMR-ULS; record Bagnoli-IrpinoEW).

Figure 5.5.2.6. $S_{a,e}$ vs. $\Delta_{max}/h$ and $\Delta_e/h$: influence of energy-controlled strength degradation (frame PMR-ULS; record Bagnoli-IrpinoNS).
Figure 5.5.2.7. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_{r}/h$: influence of energy-controlled strength degradation (frame PMR-ULS; record SturnoEW).

Figure 5.5.2.8. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_{r}/h$: influence of energy-controlled strength degradation (frame PMR-ULS; record SturnoNS).
Figure 5.5.2.9. $S_{a,e}$ vs. Damage Index: influence of strength degradation (frame PMR-SLS; record Bagnoli-IrpinoEW).

Figure 5.5.2.10. $S_{a,e}$ vs. Damage Index: influence of strength degradation (frame PMR-SLS; record Bagnoli-IrpinoNS).
Figure 5.5.2.11. $S_{a,e}$ vs. Damage Index: influence of strength degradation (frame PMR-SLS; record Bagnoli-IrpinoEW).

Figure 5.5.2.12. $S_{a,e}$ vs. Damage Index: influence of strength degradation (frame PMR-SLS; record Bagnoli-IrpinoEW).
Figure 5.5.2.13. $S_{a,e}$ vs. Damage Index: influence of strength degradation (frame PMR-ULS; record Bagnoli-IrpinoEW).

Figure 5.5.2.14. $S_{a,e}$ vs. Damage Index: influence of strength degradation (frame PMR-ULS; record Bagnoli-IrpinoEW).
Figure 5.5.2.15. $S_{a,e}$ vs. Damage Index: influence of strength degradation (frame PMR-ULS; record Bagnoli-IrpinoEW).

Figure 5.5.2.16. $S_{a,e}$ vs. Damage Index: influence of strength degradation (frame PMR-ULS; record Bagnoli-IrpinoEW).
Figure 5.5.2.17. $S_{a,e}$ vs. $\Delta_{max}/h$ and $\Delta_r/h$: influence of deformation-controlled strength degradation (frame PMR-SLS; record Bagnoli-IrpinoEW).

Figure 5.5.2.18. $S_{a,e}$ vs. $\Delta_{max}/h$ and $\Delta_r/h$: influence of deformation-controlled strength degradation (frame PMR-SLS; Bagnoli-IrpinoNS record).
Figure 5.5.2.19. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_r/h$: influence of deformation-controlled strength degradation (frame PMR-SLS; record SturnoEW).

Figure 5.5.2.20. $S_{a,e}$ vs. $\Delta_{\text{max}}/h$ and $\Delta_r/h$: influence of deformation-controlled strength degradation (frame PMR-SLS; record SturnoNS).
Figure 5.5.2.21. $S_{a,e}$ vs. $\Delta_{max}/h$ and $\Delta_r/h$: influence of deformation-controlled strength degradation (frame PMR-ULS; record Bagnoli-IrpinoEW).

Figure 5.5.2.22. $S_{a,e}$ vs. $\Delta_{max}/h$ and $\Delta_r/h$: influence of deformation-controlled strength degradation (frame PMR-ULS; Bagnoli-IrpinoNS record).
Figure 5.5.2.23. $S_{a,e}$ vs. $\Delta_{max}/h$ and $\Delta_r/h$: influence of deformation-controlled strength degradation (frame PMR-ULS; record SturnoEW).

Figure 5.5.2.24. $S_{a,e}$ vs. $\Delta_{max}/h$ and $\Delta_r/h$: influence of deformation-controlled strength degradation (frame PMR-ULS; record SturnoNS).
Conclusions and further Developments

This dissertation has been mainly focused on the identification of the conditions leading to collapse ordinary structures, i.e. structures designed to suffer damage to some extent during strong earthquakes.

The practical concept of Seismic Stability as the measure of the structure ability to continue to serve, after the earthquake, the function it was designed for before the earthquake, has been introduced. Consequently, the Seismic Stability Limit State (collapse) has been identified, qualitatively, as that limiting damage condition for which the structure looses its ability to sustain gravity loads after the earthquake. The need for defining this Limit State from an engineering point of view, i.e. introducing an objective and rational tool for identifying its achievement, has been emphasised. Definition of such a tool has been the main objective of the whole dissertation.

The objective has been pursued firstly studying the inelastic response of single degree of freedom (SDoF) systems. Based on the idea that measuring the Seismic Stability of the structure is equivalent to measure the stability of the post-earthquake mechanical state of the structure, the lateral plastic displacement capacity of the system has been defined and the way of computing it identified in Chapter 2.

In Chapter 3, the approach developed with reference to SDoF systems has been extended to study the Seismic Stability of multi-degree of freedom (MDoF) structures.

Once defined the Limit State, it has been possible to define also a 'collapse-based' global damage index, i.e. a number ranging from 0 (no damage) to 1 (collapse). The proposed Damage Index could be the required engineering tool
for evaluating the degree of Seismic Stability of the structure and, at the limit condition, for identifying the achievement of the Seismic Stability Limit State.

Finally, in Chapter 5 several results of a numerical study of MR steel frames have been presented. These results confirmed the main conclusions qualitatively drawn in previous Chapters and the validity of the proposed global damage evaluation approach. In particular, it has been emphasised the paramount role of strength degradation in determining the earthquake intensity producing the achievement of the Seismic Stability Limit State. Besides, the importance of the achievement of high-level performance objectives (Serviceability requirements) for the structure response under rare or very rare earthquakes, i.e. at low levels of performances (Life-Safety and Structural Stability), has been highlighted.

The problem of evaluating the Seismic Stability of structures is quite complex. This difficulty mainly arises from the need to describe degradation phenomena of those structural components engaged in the plastic range of deformation during the earthquake. This is a problem by itself, which requires a very deep research. Besides, in case of SDoF systems, strength degradation is the origin of the correlation between deformation demand and deformation capacity. In case of MDoF structures, the additional effect of multiple modes of vibrations, which could combine between them in different manners when increasing the earthquake intensity, further complicates the problem of the a-priori prediction of the deformation capacity. Then, at the current state of knowledge, the numerical approach, based on the step-by-step numerical integration of the equations of motion, appears to be the only rational way to face the problem.

Moreover, it is not yet well understood which is the correct way for measuring the ground motion damage potential. The use of either the first-mode elastic spectral acceleration or the peak ground acceleration, which are the only dependable seismological parameters currently available, has shown several limitations. In particular, it has been highlighted that these two parameters do not always give rise to a monotonically increasing damage to the structure.
References


